Exercise walk-through

8.7
Volumes with Cross Sections: Squares and Rectangles

AP Calculus BC

You must be able to

- write the **side length** of a slice as the gap between two curves
- integrate the **cross-sectional area** : $V = \int_a^b A(x)\,dx$
- handle a square ($A = s^2$) or a rectangle cross section

1 The method

Method — Volume as an integral of area

A solid whose cross section perpendicular to the x-axis has area $A(x)$ has volume

$$V = \int_a^b A(x)\,dx.$$

"Integrate the cross-sectional area" is the whole method.

AP Calculus BC

Exercises 2.1–2.3

2.1 A slice has area $A(x) = x^2$. Write the volume integral over $[0, 3]$. [1]

2.2 A solid has square cross sections with side $s = 2x$ on $[0, 2]$. Find its volume. [3]

2.3 Evaluate $\displaystyle\int_0^1 (1 - x)^2 \, dx$. [2]

Exercises 2.1–2.3

2.1 A slice has area $A(x) = x^2$. Write the volume integral over $[0, 3]$. [1]

2.1

- $\displaystyle\int_0^3 x^2\,dx$

2.2 A solid has square cross sections with side $s = 2x$ on $[0, 2]$. Find its volume. [3]

2.2

- $A = (2x)^2 = 4x^2$; $\displaystyle\int_0^2 4x^2\,dx = \left[\tfrac{4x^3}{3}\right]_0^2 = \tfrac{32}{3}$

2.3 Evaluate $\displaystyle\int_0^1 (1 - x)^2\,dx$. [2]

2.3

- $\displaystyle\int_0^1 (1-x)^2\,dx = \left[-\tfrac{(1-x)^3}{3}\right]_0^1 = 0 - (-\tfrac{1}{3}) = \tfrac{1}{3}$

Exercise 3.1

3.1 The volume of a solid with cross-sectional area $A(x)$ is [1]

- **A** $\displaystyle\int_a^b \sqrt{A(x)}\,dx$
- **B** $\displaystyle\int_a^b A(x)\,dx$
- **C** $\displaystyle\pi\int_a^b A(x)\,dx$
- **D** $\displaystyle\int_a^b A(x)^2\,dx$

Exercise 3.1

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3.1 B

- integrate the cross-sectional area

Exercise 3.2 (a)

3.2 The base of a solid is the region between $y = 4 - x^2$ and the x-axis. Cross sections perpendicular to the x-axis are squares.

(a) Write the side length s as a function of x. [1]

Exercise 3.2 (a)

3.2 The base of a solid is the region between $y = 4 - x^2$ and the x-axis. Cross sections perpendicular to the x-axis are squares.

(a) Write the side length s as a function of x. [1]

(a)

- $s = 4 - x^2$

Exercise 3.2 (b)

3.2 The base of a solid is the region between $y = 4 - x^2$ and the x-axis. Cross sections perpendicular to the x-axis are squares.

(b) Set up the volume integral (with limits). [2]

Exercise 3.2 (b)

3.2 The base of a solid is the region between $y = 4 - x^2$ and the x-axis. Cross sections perpendicular to the x-axis are squares.

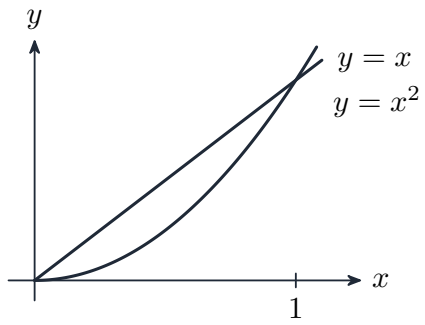
(b) Set up the volume integral (with limits). [2]

(b)

- $\int_{-2}^{2} (4 - x^2)^2 \, dx$

Exercise 3.3

3.3 The base of a solid is bounded by $y = x$ and $y = x^2$. Cross sections perpendicular to the x-axis are squares with side equal to the gap between the curves. Set up and evaluate the volume. [5]



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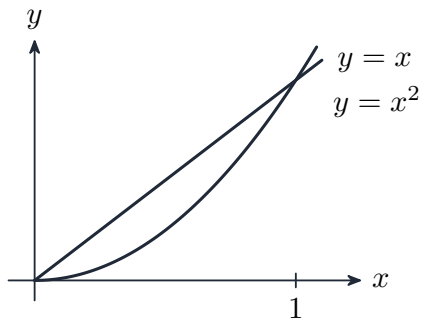

3.3

- $s = x - x^2$ on $[0, 1]$; $V = \displaystyle\int_0^1 (x - x^2)^2\,dx = \int_0^1 (x^2 - 2x^3 + x^4)\,dx = \left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right]_0^1$ $= \frac{1}{3} - \frac{1}{2} + \frac{1}{5} = \frac{1}{30}$