

Exercise walk-through

8.11

Volume with Washer Method: Revolving Around the x - or y -Axis

AP Calculus BC

Volumes with known cross sections

If a solid has a known **cross section** perpendicular to an axis, the volume is the integral of its **area**:

$$V = \int_a^b A(x) dx.$$

The exam phrase “the base of a solid... cross sections are squares” signals exactly this.

square of side s : $A = s^2$

rectangle, height ks : $A = ks^2$

equilateral **triangle**: $A = \frac{\sqrt{3}}{4}s^2$

semicircle, diameter s : $A = \frac{\pi}{8}s^2$

Set the shape's key length equal to the slice length $s(x) = f(x) - g(x)$, then integrate.

Disc, washer and shell

Disc method

Rotating a region that touches the axis: $V = \pi \int R(x)^2 dx$ – each slice is a solid circle.

Washer method

The **washer method** is for a region with a gap: $V = \pi \int (R^2 - r^2) dx$, outer radius minus inner.

Which variable

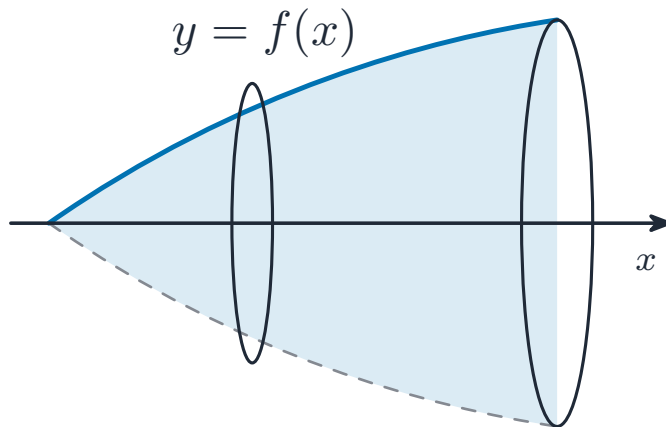
Slice perpendicular to the axis of rotation. Rotating about a horizontal axis integrates in x ; a vertical axis, in y .

A shifted axis

Rotating about $y = k$ changes the radii to $|f(x) - k|$ – the method is unchanged.

Sketch the slice before writing the integral. Radius, thickness and limits all come from the picture.

Volume with Disc Method: Revolving Around the x - or y -Axis



rotate the region around the x -axis to sweep a solid

Rotating a region about an axis sweeps out a solid of revolution

Exercises 2.1·2.3

2.1 State the washer-method formula for the volume.

[1]

2.2 For $R = 4$ and $r = 2$, find $\pi(R^2 - r^2)$.

[1]

2.3 Evaluate $\pi \int_0^1 (x - x^2) dx$.

[2]

Exercises 2.1-2.3

2.1 State the washer-method formula for the volume.

[1]

2.1

- $$V = \pi \int_a^b (R_{\text{outer}}^2 - R_{\text{inner}}^2) dx$$

2.2 For $R = 4$ and $r = 2$, find $\pi(R^2 - r^2)$.

[1]

2.2

- $$\pi(16 - 4) = 12\pi$$

2.3 Evaluate $\pi \int_0^1 (x - x^2) dx$.

[2]

2.3

- $$\pi \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \pi \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{\pi}{6}$$

Exercise 3.1

3.1 The washer method integrates

[1]

- ☐ **A** $\pi \int (R - r)^2 dx$
- B** $\pi \int (R^2 - r^2) dx$
- C** $\pi \int R^2 dx$
- D** $\int (R^2 - r^2) dx$

Exercise 3.1

3.1 The washer method integrates

[1]

- ☐ **A** $\pi \int (R - r)^2 dx$
- B** $\pi \int (R^2 - r^2) dx$
- C** $\pi \int R^2 dx$
- D** $\int (R^2 - r^2) dx$

3.1 B

- subtract the **squares** of the radii

Exercise 3.2 (a)

3.2 The region between $y = x$ and $y = x^2$ (from $x = 0$ to $x = 1$) is revolved about the x -axis.

(a) Identify the outer and inner radii.

[2]

Exercise 3.2 (a)

3.2 The region between $y = x$ and $y = x^2$ (from $x = 0$ to $x = 1$) is revolved about the x -axis.

(a) Identify the outer and inner radii.

[2]

(a)

- On $(0, 1)$, $y = x$ is above $y = x^2$; outer $R = x$, inner $r = x^2$

Exercise 3.2 (b)

3.2 The region between $y = x$ and $y = x^2$ (from $x = 0$ to $x = 1$) is revolved about the x -axis.

(b) Set up and evaluate the volume.

[4]

Exercise 3.2 (b)

3.2 The region between $y = x$ and $y = x^2$ (from $x = 0$ to $x = 1$) is revolved about the x -axis.

(b) Set up and evaluate the volume.

[4]

(b)

- $$V = \pi \int_0^1 (x^2 - x^4) dx = \pi \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_0^1 = \pi \left(\frac{1}{3} - \frac{1}{5} \right) = \frac{2\pi}{15}$$

Exercise 3.3

3.3 The region bounded by $y = 4$ and $y = x^2$ (for $0 \leq x \leq 2$) is revolved about the x -axis. Set up the washer integral for the volume. [3]

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3.3 The region bounded by $y = 4$ and $y = x^2$ (for $0 \leq x \leq 2$) is revolved about the x -axis. Set up the washer integral for the volume. [3]

3.3

- Outer $R = 4$, inner $r = x^2$; $V = \pi \int_0^2 (4^2 - (x^2)^2) dx = \pi \int_0^2 (16 - x^4) dx$