

Exercise walk-through

# 8.1

## Finding the Average Value of a Function on an Interval

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AP Calculus BC

# Average value of a function

The **average value** over  $[a, b]$ :

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx.$$

It is the **constant height** of a rectangle on  $[a, b]$  with the **same area** as under  $f$ .

Do **not** confuse it with the **average rate of change** (which divides the *change in  $f$*  by the interval).

**Average value of  $f(x) = x^2$  on  $[0, 3]$**

$$\frac{1}{3} \int_0^3 x^2 dx = \frac{1}{3} \left[ \frac{x^3}{3} \right]_0^3 = 3.$$

# Connecting Position, Velocity, and Acceleration Using

For straight-line motion, integration reverses differentiation:

$$v(t) = \int a(t) dt, \quad s(t) = \int v(t) dt.$$

Two key distinctions: ↷

## Exercises 2.1-2.3

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**2.1** Write the formula for the average value of  $f$  on  $[2, 6]$ . [1]

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**2.2** Find the average value of  $f(x) = 4x$  on  $[0, 5]$ . [3]

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**2.3** Find the average value of  $f(x) = 6x^2$  on  $[0, 2]$ . [3]

## Exercises 2.1-2.3

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**2.1** Write the formula for the average value of  $f$  on  $[2, 6]$ . [1]

**2.1**

- $$\frac{1}{6-2} \int_2^6 f(x) \, dx$$

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**2.2** Find the average value of  $f(x) = 4x$  on  $[0, 5]$ . [3]

**2.2**

- $$\frac{1}{5} \int_0^5 4x \, dx = \frac{1}{5} [2x^2]_0^5 = \frac{1}{5}(50) = 10$$

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**2.3** Find the average value of  $f(x) = 6x^2$  on  $[0, 2]$ . [3]

**2.3**

- $$\frac{1}{2} \int_0^2 6x^2 \, dx = \frac{1}{2} [2x^3]_0^2 = \frac{1}{2}(16) = 8$$

## Exercise 3.1

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**3.1** The average value of  $f$  on  $[a, b]$  is

[1]

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**A**  $f(b) - f(a)$

**B**  $\frac{f(b) - f(a)}{b - a}$

**C**  $\frac{1}{b - a} \int_a^b f \, dx$

**D**  $\int_a^b f \, dx$

## Exercise 3.1

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**3.1** The average value of  $f$  on  $[a, b]$  is

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**D**  $\int_a^b f \, dx$

**3.1 C**

- the average value is the integral divided by the width

## Exercise 3.2 (a)

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**3.2** A tank's inflow rate is  $r(t) = 12 - t$  litres/min for  $0 \leq t \leq 8$ .

(a) Find the average inflow rate over this interval.

[3]

## Exercise 3.2 (a)

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**3.2** A tank's inflow rate is  $r(t) = 12 - t$  litres/min for  $0 \leq t \leq 8$ .

(a) Find the average inflow rate over this interval.

[3]

(a)

$$\bullet \quad \frac{1}{8} \int_0^8 (12 - t) dt = \frac{1}{8} \left[ 12t - \frac{t^2}{2} \right]_0^8 = \frac{1}{8} (96 - 32) = 8$$

## Exercise 3.2 (b)

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**3.2** A tank's inflow rate is  $r(t) = 12 - t$  litres/min for  $0 \leq t \leq 8$ .

(b) State the units of your answer.

[1]

## Exercise 3.2 (b)

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**3.2** A tank's inflow rate is  $r(t) = 12 - t$  litres/min for  $0 \leq t \leq 8$ .

(b) State the units of your answer.

[1]

(b)

- litres per minute

## Exercise 3.3

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**3.3** The temperature over a day is modelled by  $T(t) = 20 + 6t - t^2$  for  $0 \leq t \leq 6$  hours. Find the average temperature over the interval. [4]

## Exercise 3.3

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**3.3** The temperature over a day is modelled by  $T(t) = 20 + 6t - t^2$  for  $0 \leq t \leq 6$  hours. Find the average temperature over the interval. [4]

**3.3**

$$\bullet \quad \frac{1}{6} \int_0^6 (20 + 6t - t^2) dt = \frac{1}{6} \left[ 20t + 3t^2 - \frac{t^3}{3} \right]_0^6 = \frac{1}{6} (120 + 108 - 72) = \frac{1}{6} (156) = 26^\circ$$