

Exercise walk-through

7.9

Logistic Models with Differential Equations

AP Calculus BC

Logistic growth (BC only)

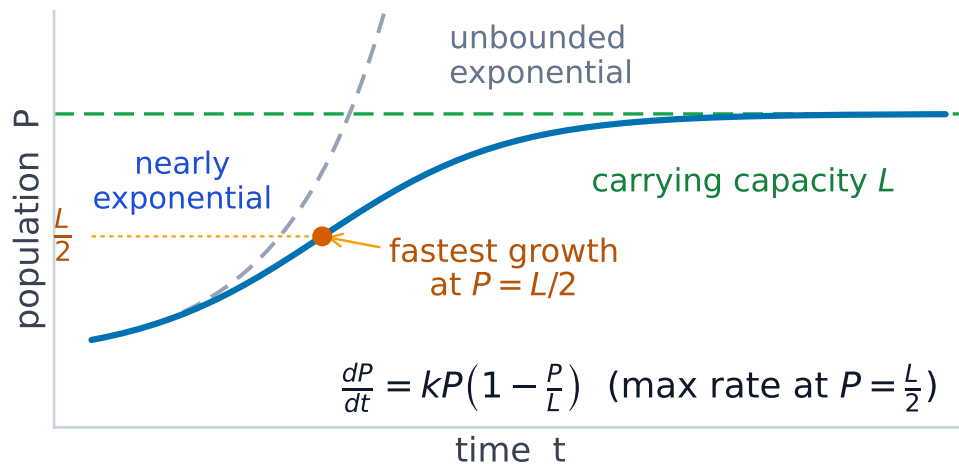
Real populations cannot grow for ever. The **logistic carrying capacity** L :

model adds a ceiling, the

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{L} \right).$$

Growth is fastest at $P = \frac{L}{2}$, and $P \rightarrow L$ as $t \rightarrow \infty$.

Logistic growth (BC only), in detail



logistic: $dP/dt = kP(1 - P/L)$ · $P \rightarrow L$ as $t \rightarrow \infty$

Real populations cannot grow for ever.

Exponential Models with Differential Equations



A cooling cup of coffee: Newton's law of cooling is a classic differential equation model

Exam tips

- Solve a **separable** equation by getting all y on one side and all x on the other, then integrating both sides (add $+C$ once).
- Use the initial condition to find C (a particular solution).
- Sketch or read a **slope field**: the little segments show $\frac{dy}{dx}$ at each point, and a solution curve follows them.
- Recognise **exponential** models $\frac{dy}{dt} = ky \Rightarrow y = Ce^{kt}$ (growth/decay).
- A differential equation gives the slope – you must integrate to recover the function.

Exercises 2.1·2.3

2.1 For $\frac{dP}{dt} = 0.5P\left(1 - \frac{P}{2000}\right)$, state the carrying capacity. [1]

2.2 For the same model, state $\lim_{t \rightarrow \infty} P$ (given $P(0) = 100$). [1]

2.3 At what population is growth fastest for that model? [1]

Exercises 2.1·2.3

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2.1

- $L = 2000$

2.2 For the same model, state $\lim_{t \rightarrow \infty} P$ (given $P(0) = 100$). [1]

2.2

- 2000

2.3 At what population is growth fastest for that model? [1]

2.3

- $P = 1000$ (i.e. $L/2$)

Exercise 3.1

3.1 In $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$, growth is fastest when $P =$

[1]

☐**A** 0**B** $\frac{L}{4}$ **C** $\frac{L}{2}$ **D** L

Exercise 3.1

3.1 In $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$, growth is fastest when $P =$ [1]

☐

A 0

B $\frac{L}{4}$

C $\frac{L}{2}$

D L

3.1 C

- fastest growth at the inflection point $P = \frac{L}{2}$

Exercise 3.2 (a)

3.2 A fish population follows $\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{5000}\right)$, with $P(0) = 500$.

(a) State the carrying capacity.

[1]

Exercise 3.2 (a)

3.2 A fish population follows $\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{5000}\right)$, with $P(0) = 500$.

(a) State the carrying capacity.

[1]

(a)

- 5000

Exercise 3.2 (b)

3.2 A fish population follows $\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{5000}\right)$, with $P(0) = 500$.

(b) State $\lim_{t \rightarrow \infty} P$.

[1]

Exercise 3.2 (b)

3.2 A fish population follows $\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{5000}\right)$, with $P(0) = 500$.

(b) State $\lim_{t \rightarrow \infty} P$.

[1]

(b)

- 5000

Exercise 3.2 (c)

3.2 A fish population follows $\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{5000}\right)$, with $P(0) = 500$.

(c) Find the population at which the population grows fastest.

[1]

Exercise 3.2 (c)

3.2 A fish population follows $\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{5000}\right)$, with $P(0) = 500$.

(c) Find the population at which the population grows fastest.

[1]

(c)

- 2500

Exercise 3.3

3.3 A logistic model has carrying capacity 1200 and $P(0) = 1500$. State, with a reason, whether the population increases or decreases toward the carrying capacity. [2]

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3.3

- Decreases – $P(0) = 1500 > 1200 = L$, so $1 - \frac{P}{L} < 0$ and $\frac{dP}{dt} < 0$, pulling P down toward 1200