

Exercise walk-through

7.7

Finding Particular Solutions Using Initial Conditions and Separation of Variables

AP Calculus BC

Separation of variables

If $\frac{dy}{dx}$ factors into a function of x times a function of y , split them and integrate:

$$\frac{dy}{dx} = g(x)h(y) \implies \int \frac{dy}{h(y)} = \int g(x) dx.$$

Add the constant of integration **once**
– on the x -side.

all y one side,
all x the other



integrate both sides



the **general so-**
lution

Worked example – separate, integrate, then pin it down

Solve $\frac{dy}{dx} = xy$ **with** $y(0) = 2$

$$\int \frac{dy}{y} = \int x dx \Rightarrow \ln |y| = \frac{x^2}{2} + C$$

$$\Rightarrow y = Ae^{x^2/2}$$

Applying $y(0) = 2$ gives $A = 2$:

$$y = 2e^{x^2/2}.$$

An **initial condition**

(x_0, y_0) picks **one** curve from the family – there is exactly **one** solution through a given point.

From the family to the particular solution

The constant gives a **family** of curves; the initial condition selects the **particular solution** .

Watch **domain restrictions** : keep the **branch containing the initial point**
(e.g. the right sign of a square root).

Euler's method, and what it is good for

the idea

step along the slope field: $y_{n+1} = y_n + f(x_n, y_n) \Delta x$

the exam skill

carry out **two or three steps by hand** from a table – that is what is asked

the tabulation

columns for x , y , the slope, and Δy ; one row per step

what it produces

an **approximation**, not a solution

the error

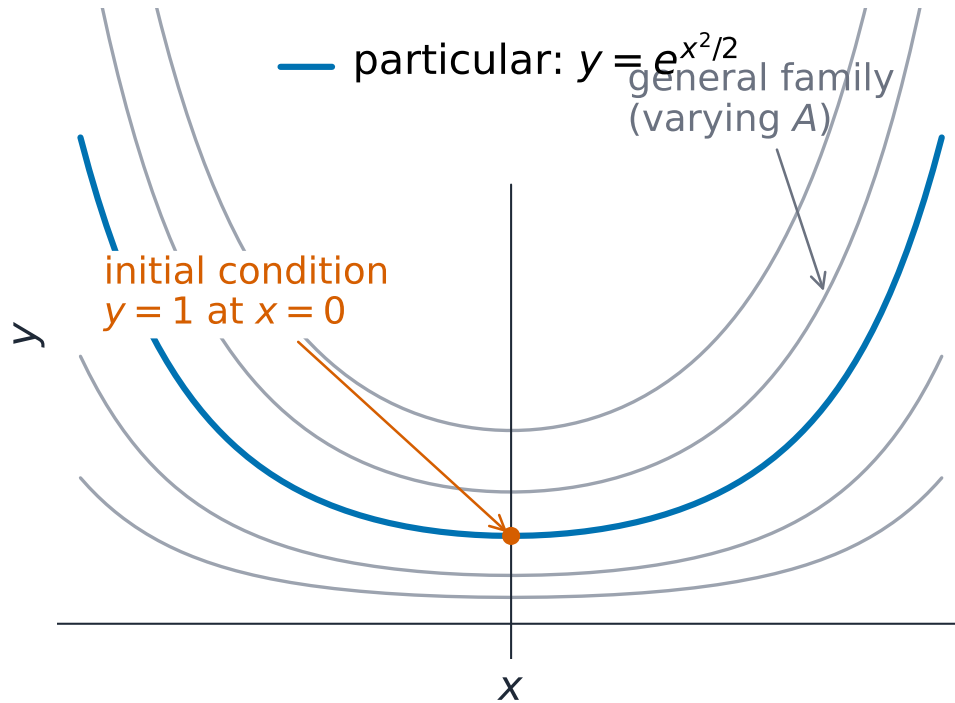
Euler's method **underestimates** a concave-up solution and overestimates a concave-down one

smaller steps

reduce the error, at the cost of more arithmetic

The concavity argument is the same one as for a tangent-line approximation – and for the same reason: both follow a straight line away from a curving one.

From the family to the particular solution, in detail



The constant gives a **family** of curves; the initial condition selects the **particular solution**.

Exercises 2.1·2.3

2.1 Solve $\frac{dy}{dx} = 4x$ with $y(1) = 5$. [3]

2.2 For $y = Ae^{2t}$ with $y(0) = 7$, find A . [1]

2.3 Solve $\frac{dy}{dx} = 3x^2$ with $y(0) = -1$. [3]

Exercises 2.1-2.3

2.1 Solve $\frac{dy}{dx} = 4x$ with $y(1) = 5$. [3]

2.1

- $y = 2x^2 + C$; $5 = 2 + C \Rightarrow C = 3$; $y = 2x^2 + 3$
-

2.2 For $y = Ae^{2t}$ with $y(0) = 7$, find A . [1]

2.2

- $A = 7$
-

2.3 Solve $\frac{dy}{dx} = 3x^2$ with $y(0) = -1$. [3]

2.3

- $y = x^3 + C$; $-1 = 0 + C \Rightarrow C = -1$; $y = x^3 - 1$

Exercise 3.1

3.1 The general solution $y = x^2 + C$ passes through $(2, 10)$. Then $C =$ [1]

☐

A 2

B 6

C 10

D 14

Exercise 3.1

3.1 The general solution $y = x^2 + C$ passes through $(2, 10)$. Then $C =$ [1]

☐

A 2

B 6

C 10

D 14

3.1 B

- $10 = 4 + C \Rightarrow C = 6$

Exercise 3.2 (a)

3.2 A population satisfies $\frac{dP}{dt} = 0.5P$ with $P(0) = 200$.

(a) Write the general solution.

[2]

Exercise 3.2 (a)

3.2 A population satisfies $\frac{dP}{dt} = 0.5P$ with $P(0) = 200$.

(a) Write the general solution.

[2]

(a)

- $P = Ae^{0.5t}$

Exercise 3.2 (b)

3.2 A population satisfies $\frac{dP}{dt} = 0.5P$ with $P(0) = 200$.

(b) Find the particular solution using the initial condition.

[1]

Exercise 3.2 (b)

3.2 A population satisfies $\frac{dP}{dt} = 0.5P$ with $P(0) = 200$.

(b) Find the particular solution using the initial condition.

[1]

(b)

- $P(0) = A = 200$, so $P = 200e^{0.5t}$

Exercise 3.2 (c)

3.2 A population satisfies $\frac{dP}{dt} = 0.5P$ with $P(0) = 200$.

(c) Predict P at $t = 4$.

[2]

Exercise 3.2 (c)

3.2 A population satisfies $\frac{dP}{dt} = 0.5P$ with $P(0) = 200$.

(c) Predict P at $t = 4$.

[2]

(c)

- $P(4) = 200e^2 \approx 1478$

Exercise 3.3

3.3 Solve $\frac{dy}{dx} = \frac{6x}{y}$ with $y(0) = 4$, giving y explicitly.

[4]

Exercise 3.3

3.3 Solve $\frac{dy}{dx} = \frac{6x}{y}$ with $y(0) = 4$, giving y explicitly.

[4]

3.3

- $y \, dy = 6x \, dx$; $\frac{1}{2}y^2 = 3x^2 + C$; at $(0, 4)$: $8 = C$; $y = \sqrt{6x^2 + 16}$