

Exercise walk-through

7.5

Approximating Solutions Using Euler's Method

AP Calculus BC

What we will cover

- 1 Modelling with differential equations
- 2 Verifying a solution
- 3 Slope fields
- 4 Separation of variables
- 5 Particular solutions
- 6 Exponential growth & decay
- 7 Logistic growth

$$\frac{dy}{dt} = ky$$



$$y = y_0 e^{kt}$$

Approximating Solutions Using Euler's Method

Euler's method
slope field.

approximates a solution numerically by stepping along the

Starting from a known point, take a small step Δx and update:

$$y_{\text{new}} = y_{\text{old}} + \frac{dy}{dx} \cdot \Delta x.$$

Repeat.

Euler's method, and what it is good for

the idea

step along the slope field: $y_{n+1} = y_n + f(x_n, y_n) \Delta x$

the exam skill

carry out **two or three steps by hand** from a table – that is what is asked

the tabulation

columns for x , y , the slope, and Δy ; one row per step

what it produces

an **approximation**, not a solution

the error

Euler's method **underestimates** a concave-up solution and overestimates a concave-down one

smaller steps

reduce the error, at the cost of more arithmetic

The concavity argument is the same one as for a tangent-line approximation – and for the same reason: both follow a straight line away from a curving one.

Exercises 2.1·2.3

2.1 State the Euler update formula.

[1]

2.2 For $\frac{dy}{dx} = 2x$, $y(1) = 3$, $\Delta x = 0.5$, find y at $x = 1.5$ (one step).

[2]

2.3 Continue: from $(1.5, y)$ take one more step to estimate $y(2)$.

[2]

Exercises 2.1·2.3

2.1 State the Euler update formula.

[1]

2.1

- $y_{\text{new}} = y_{\text{old}} + \frac{dy}{dx} \cdot \Delta x$

2.2 For $\frac{dy}{dx} = 2x$, $y(1) = 3$, $\Delta x = 0.5$, find y at $x = 1.5$ (one step).

[2]

2.2

- slope at $(1, 3) = 2$; $y \approx 3 + 2(0.5) = 4$

2.3 Continue: from $(1.5, y)$ take one more step to estimate $y(2)$.

[2]

2.3

- slope at $(1.5, 4) = 3$; $y \approx 4 + 3(0.5) = 5.5$

Exercises 3.1-3.2

3.1 Euler's method approximates a solution using

[1]

☐

A the exact antiderivative

B tangent-line steps along the slope field

C a Riemann sum

D the second derivative

3.2 Given $\frac{dy}{dx} = x - y$ with $y(0) = 2$ and $\Delta x = 0.5$, estimate $y(1)$ using two Euler steps.
Show the table.

[4]

Exercises 3.1-3.2

3.1 Euler's method approximates a solution using

[1]

☐

A the exact antiderivative

B tangent-line steps along the slope field

C a Riemann sum

D the second derivative

3.1 B

- it steps along tangent lines given by the slope field

3.2 Given $\frac{dy}{dx} = x - y$ with $y(0) = 2$ and $\Delta x = 0.5$, estimate $y(1)$ using two Euler steps. Show the table. [4]

3.2

- Step 1 at $(0, 2)$: slope $0 - 2 = -2$; $y \approx 2 + (-2)(0.5) = 1$ at $x = 0.5$. Step 2 at $(0.5, 1)$: slope $0.5 - 1 = -0.5$; $y \approx 1 + (-0.5)(0.5) = 0.75$ at $x = 1$

Exercise 3.3

3.3 For a solution that is concave up, state whether Euler's method over- or under-estimates the true value, and why. [2]

Exercise 3.3

3.3 For a solution that is concave up, state whether Euler's method over- or under-estimates the true value, and why. [2]

3.3

- Under-estimate – for a concave-up curve each tangent-line step lies below the true curve