

Exercise walk-through

7.4

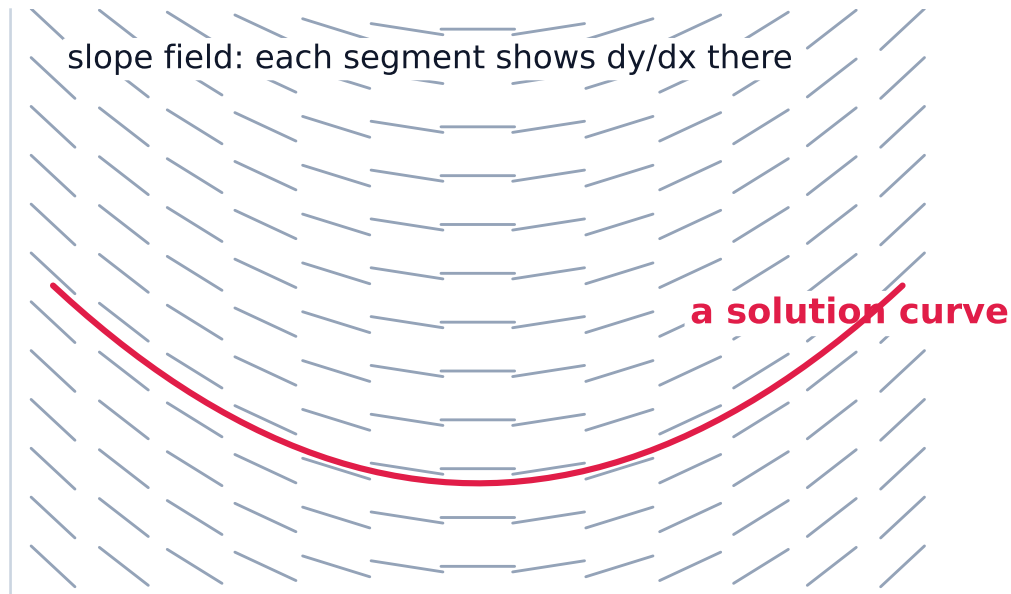
Reasoning Using Slope Fields

AP Calculus BC

Slope fields

A **slope field** draws the equation as a grid of short segments: at each (x, y) the segment has slope $\frac{dy}{dx}$ there.

It shows the **shape** of the solution curves **without solving**.



Approximating Solutions Using Euler's Method

Euler's method
slope field.

approximates a solution numerically by stepping along the

Starting from a known point, take a small step Δx and update:

$$y_{\text{new}} = y_{\text{old}} + \frac{dy}{dx} \cdot \Delta x.$$

Repeat.

Reading a slope field

segments **flat** $\Rightarrow \frac{dy}{dx} = 0$: the solution is momentarily **level**

segments **steepen** $\Rightarrow$ the solution rises or falls **faster**

a **horizontal row** of equal slopes $\Rightarrow$ the rate depends only on y

Exam skill: start at the given point and **follow** the segments

A solution curve stays **tangent** to the field everywhere.

Exercises 2.1·2.3

2.1 Find the equilibrium solution of $\frac{dy}{dx} = y - 5$. [1]

2.2 For $\frac{dy}{dx} = 3 - y$, state the equilibrium value of y . [1]

2.3 For $\frac{dy}{dx} = y - 5$, if a solution starts at $y = 6$, does it move toward or away from $y = 5$? [1]

Exercises 2.1-2.3

2.1 Find the equilibrium solution of $\frac{dy}{dx} = y - 5$. [1]

2.1

- $y = 5$
-

2.2 For $\frac{dy}{dx} = 3 - y$, state the equilibrium value of y . [1]

2.2

- $y = 3$
-

2.3 For $\frac{dy}{dx} = y - 5$, if a solution starts at $y = 6$, does it move toward or away from $y = 5$? [1]

2.3

- Away – at $y = 6$, $\frac{dy}{dx} = 1 > 0$, so y increases, moving away from 5

Exercise 3.1

3.1 An equilibrium solution of a differential equation is a value of y where [1]

- ☐ **A** $\frac{dy}{dx} = 0$
- B** $y = 0$
- C** $x = 0$
- D** $\frac{dy}{dx}$ is undefined

Exercise 3.1

3.1 An equilibrium solution of a differential equation is a value of y where [1]

- ☐ **A** $\frac{dy}{dx} = 0$
B $y = 0$
C $x = 0$
D $\frac{dy}{dx}$ is undefined

3.1 A

- an equilibrium has $\frac{dy}{dx} = 0$

Exercise 3.2 (a)

3.2 Consider $\frac{dy}{dx} = 2 - y$.

(a) Find the equilibrium solution.

[1]

Exercise 3.2 (a)

3.2 Consider $\frac{dy}{dx} = 2 - y$.

(a) Find the equilibrium solution.

[1]

(a)

- $y = 2$

Exercise 3.2 (b)

3.2 Consider $\frac{dy}{dx} = 2 - y$.

(b) For a solution starting at $y = 5$, state whether y increases or decreases, with a reason. [2]

Exercise 3.2 (b)

3.2 Consider $\frac{dy}{dx} = 2 - y$.

(b) For a solution starting at $y = 5$, state whether y increases or decreases, with a reason. [2]

(b)

- Decreases – at $y = 5$, $\frac{dy}{dx} = 2 - 5 = -3 < 0$

Exercise 3.2 (c)

3.2 Consider $\frac{dy}{dx} = 2 - y$.

(c) State $\lim_{x \rightarrow \infty} y$ for that solution.

[1]

Exercise 3.2 (c)

3.2 Consider $\frac{dy}{dx} = 2 - y$.

(c) State $\lim_{x \rightarrow \infty} y$ for that solution.

[1]

(c)

- $y \rightarrow 2$

Exercise 3.3

3.3 A slope field for $\frac{dy}{dx} = y(4 - y)$ has two equilibrium lines. State both, and say which one solutions starting near $y = 1$ approach. [3]

Exercise 3.3

3.3 A slope field for $\frac{dy}{dx} = y(4 - y)$ has two equilibrium lines. State both, and say which one solutions starting near $y = 1$ approach. [3]

3.3

- Equilibria $y = 0$ and $y = 4$; a solution starting at $y = 1$ has $\frac{dy}{dx} = 1(3) > 0$, so it rises toward $y = 4$