

Exercise walk-through

7.3

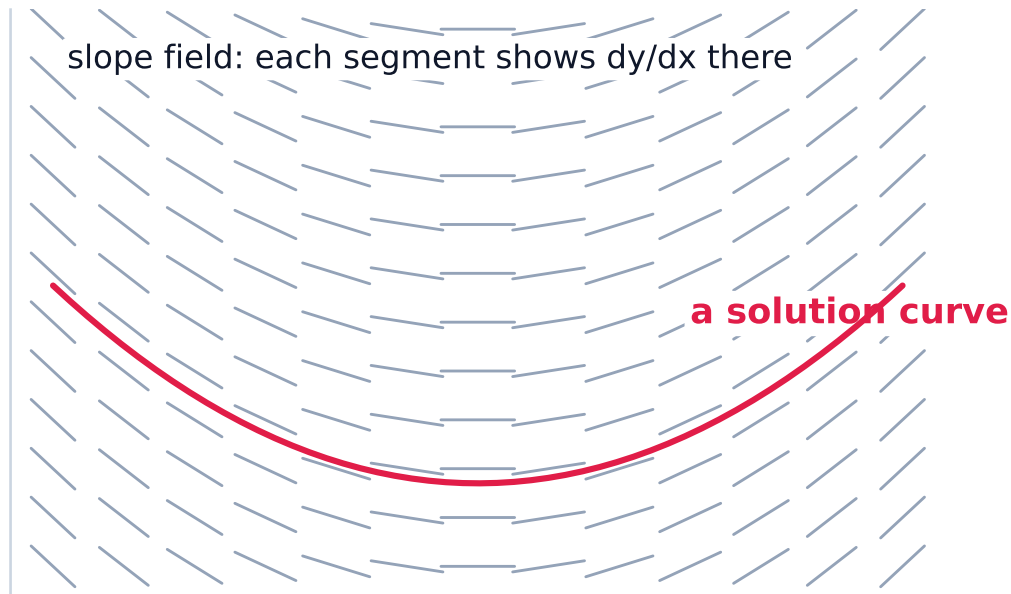
Sketching Slope Fields

AP Calculus BC

Slope fields

A **slope field** draws the equation as a grid of short segments: at each (x, y) the segment has slope $\frac{dy}{dx}$ there.

It shows the **shape** of the solution curves **without solving**.



Approximating Solutions Using Euler's Method

Euler's method
slope field.

approximates a solution numerically by stepping along the

Starting from a known point, take a small step Δx and update:

$$y_{\text{new}} = y_{\text{old}} + \frac{dy}{dx} \cdot \Delta x.$$

Repeat.

Reading a slope field

segments **flat** $\Rightarrow \frac{dy}{dx} = 0$: the solution is momentarily **level**

segments **steepen** $\Rightarrow$ the solution rises or falls **faster**

a **horizontal row** of equal slopes $\Rightarrow$ the rate depends only on y

Exam skill: start at the given point and **follow** the segments

A solution curve stays **tangent** to the field everywhere.

Exercises 2.1·2.3

2.1 For $\frac{dy}{dx} = x - y$, find the slope at $(3, 1)$. [1]

2.2 For $\frac{dy}{dx} = xy$, find the slope at $(2, -1)$. [1]

2.3 For $\frac{dy}{dx} = y - 1$, state where all the segments are **horizontal**. [1]

Exercises 2.1-2.3

2.1 For $\frac{dy}{dx} = x - y$, find the slope at $(3, 1)$. [1]

2.1

- $3 - 1 = 2$
-

2.2 For $\frac{dy}{dx} = xy$, find the slope at $(2, -1)$. [1]

2.2

- $(2)(-1) = -2$
-

2.3 For $\frac{dy}{dx} = y - 1$, state where all the segments are **horizontal**. [1]

2.3

- Where $y - 1 = 0$, i.e. along the line $y = 1$

Exercise 3.1

3.1 In the slope field for $\frac{dy}{dx} = 2x$, along the line $x = 0$ the segments are [1]

☐

A vertical

B horizontal

C slope +1

D slope +2

Exercise 3.1

3.1 In the slope field for $\frac{dy}{dx} = 2x$, along the line $x = 0$ the segments are [1]

☐

A vertical

B horizontal

C slope +1

D slope +2

3.1 B

- at $x = 0$ the slope is $2(0) = 0$, horizontal

Exercise 3.2 (a)

3.2 A slope field is given by $\frac{dy}{dx} = x + 1$.

(a) Find the slope at the points $(-1, 0)$, $(0, 2)$, and $(2, -3)$.

[3]

Exercise 3.2 (a)

3.2 A slope field is given by $\frac{dy}{dx} = x + 1$.

(a) Find the slope at the points $(-1, 0)$, $(0, 2)$, and $(2, -3)$.

[3]

(a)

- $(-1, 0)$: 0; $(0, 2)$: 1; $(2, -3)$: 3

Exercise 3.2 (b)

3.2 A slope field is given by $\frac{dy}{dx} = x + 1$.

(b) Along which vertical line are all segments horizontal?

[1]

Exercise 3.2 (b)

3.2 A slope field is given by $\frac{dy}{dx} = x + 1$.

(b) Along which vertical line are all segments horizontal?

[1]

(b)

- $x = -1$ (where $x + 1 = 0$)

Exercise 3.3

3.3 A slope field has horizontal segments only along the x -axis ($y = 0$) and gets steeper as $|y|$ grows, with the same pattern in every vertical column. Which differential equation fits: $\frac{dy}{dx} = x$, $\frac{dy}{dx} = y$, or $\frac{dy}{dx} = xy$? Explain. [2]

Exercise 3.3

3.3 A slope field has horizontal segments only along the x -axis ($y = 0$) and gets steeper as $|y|$ grows, with the same pattern in every vertical column. Which differential equation fits: $\frac{dy}{dx} = x$, $\frac{dy}{dx} = y$, or $\frac{dy}{dx} = xy$? Explain. [2]

3.3

- $\frac{dy}{dx} = y$ – the slope is 0 where $y = 0$ and grows with $|y|$, and depends on y only, so it is the same in every column