

Exercise walk-through

7.2

Verifying Solutions for Differential Equations

AP Calculus BC

What we will cover

- 1 Modelling with differential equations
- 2 Verifying a solution
- 3 Slope fields
- 4 Separation of variables
- 5 Particular solutions
- 6 Exponential growth & decay
- 7 Logistic growth

$$\frac{dy}{dt} = ky$$



$$y = y_0 e^{kt}$$

Verifying a proposed solution

A **solution** is a **function** that makes the equation true. To verify one: **differentiate it** and substitute – if both sides match, it is a solution.

A differential equation usually has **infinitely many** solutions – a whole **family**, differing by a constant.

Check $y = 2e^{x^2/2}$ **solves** $\frac{dy}{dx} = xy$

$$\frac{dy}{dx} = 2e^{x^2/2} \cdot x = x(2e^{x^2/2}) = xy.$$

True.

The exponential model, and where it fails

The equation

Exponential growth or decay is the solution of $\frac{dy}{dt} = ky$: rate proportional to amount, giving $y = y_0 e^{kt}$.

Reading k

$k > 0$ grows, $k < 0$ decays, and $\ln 2/|k|$ is the doubling or half-life.

Where it fails

Nothing grows proportionally forever. The logistic model adds the factor $(1 - y/L)$, so growth slows as y approaches the carrying capacity L .

The tell

Fastest growth in a logistic model happens at $y = L/2$ – the inflection point of the curve.

Read the sentence into the differential equation: “proportional to the amount present” is $dy/dt = ky$.

Exercises 2.1-2.3

2.1 Verify that $y = e^{5x}$ satisfies $\frac{dy}{dx} = 5y$. [2]

2.2 Show that $y = 4e^{-t}$ satisfies $\frac{dy}{dt} = -y$. [2]

2.3 For $y = Ce^{3x}$, state $\frac{dy}{dx}$ in terms of y . [1]

Exercises 2.1-2.3

2.1 Verify that $y = e^{5x}$ satisfies $\frac{dy}{dx} = 5y$. [2]

2.1

- $\frac{dy}{dx} = 5e^{5x}$; this equals $5y = 5e^{5x}$, so it is a solution

2.2 Show that $y = 4e^{-t}$ satisfies $\frac{dy}{dt} = -y$. [2]

2.2

- $\frac{dy}{dt} = -4e^{-t}$; and $-y = -4e^{-t}$, so they match

2.3 For $y = Ce^{3x}$, state $\frac{dy}{dx}$ in terms of y . [1]

2.3

- $\frac{dy}{dx} = 3Ce^{3x} = 3y$

Exercise 3.1

3.1 Which function satisfies $\frac{dy}{dx} = 2y$?

[1]

☐

A $y = 2x$

B $y = x^2$

C $y = e^{2x}$

D $y = 2e^x$

Exercise 3.1

3.1 Which function satisfies $\frac{dy}{dx} = 2y$?

[1]

☐

A $y = 2x$

B $y = x^2$

C $y = e^{2x}$

D $y = 2e^x$

3.1 C

- $\frac{d}{dx}e^{2x} = 2e^{2x} = 2y$

Exercise 3.2 (a)

3.2 Consider $y = x^2 + 3x + C$.

(a) Show that $\frac{dy}{dx} = 2x + 3$.

[1]

Exercise 3.2 (a)

3.2 Consider $y = x^2 + 3x + C$.

(a) Show that $\frac{dy}{dx} = 2x + 3$.

[1]

(a)

- $\frac{dy}{dx} = 2x + 3$

Exercise 3.2 (b)

3.2 Consider $y = x^2 + 3x + C$.

(b) Find the value of C so that the solution passes through $(1, 6)$.

[2]

Exercise 3.2 (b)

3.2 Consider $y = x^2 + 3x + C$.

(b) Find the value of C so that the solution passes through $(1, 6)$.

[2]

(b)

- $6 = 1 + 3 + C \Rightarrow C = 2$

Exercise 3.3

3.3 Verify that $y = \sin(2x)$ satisfies $\frac{d^2y}{dx^2} = -4y$.

[3]

Exercise 3.3

3.3 Verify that $y = \sin(2x)$ satisfies $\frac{d^2y}{dx^2} = -4y$. [3]

3.3

- $\frac{dy}{dx} = 2 \cos(2x)$; $\frac{d^2y}{dx^2} = -4 \sin(2x)$; this equals $-4y = -4 \sin(2x)$