

Exercise walk-through

7.1

Modeling Situations with Differential Equations

AP Calculus BC

What we will cover

- 1 Modelling with differential equations
- 2 Verifying a solution
- 3 Slope fields
- 4 Separation of variables
- 5 Particular solutions
- 6 Exponential growth & decay
- 7 Logistic growth

$$\frac{dy}{dt} = ky$$

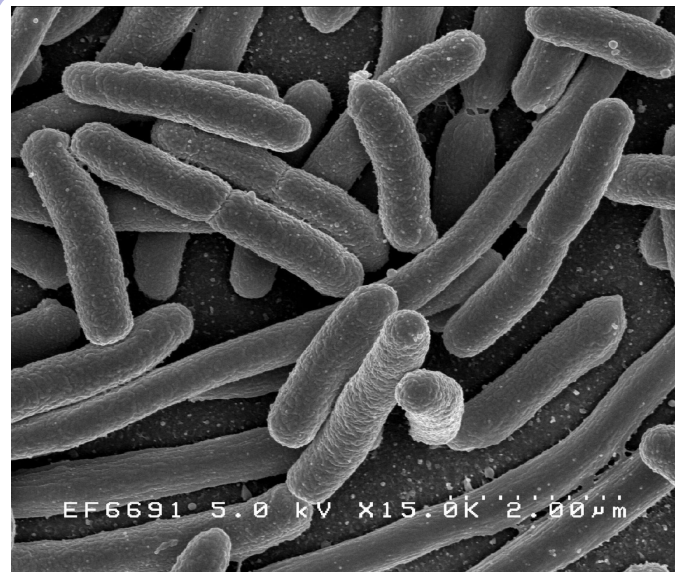


$$y = y_0 e^{kt}$$

Modelling a situation with a rate

A **differential equation** relates a function to its **own derivatives** – it describes a situation by its **rate of change**. “The rate of change of a quantity is **proportional to its size**” becomes

$$\frac{dy}{dt} = ky.$$



Bacteria growth

The exponential model, and where it fails

The equation

Exponential growth or decay is the solution of $\frac{dy}{dt} = ky$: rate proportional to amount, giving $y = y_0 e^{kt}$.

Reading k

$k > 0$ grows, $k < 0$ decays, and $\ln 2/|k|$ is the doubling or half-life.

Where it fails

Nothing grows proportionally forever. The logistic model adds the factor $(1 - y/L)$, so growth slows as y approaches the carrying capacity L .

The tell

Fastest growth in a logistic model happens at $y = L/2$ – the inflection point of the curve.

Read the sentence into the differential equation: “proportional to the amount present” is $dy/dt = ky$.

Exercises 2.1·2.3

2.1 Write a differential equation for: "the amount A decreases at a rate proportional to A ." [2]

2.2 A tank drains so that its volume V falls at a constant 3 litres per minute. Write the differential equation. [1]

2.3 Write a differential equation for: "the number of bacteria N grows at a rate proportional to N ." [1]

Exercises 2.1·2.3

2.1 Write a differential equation for: "the amount A decreases at a rate proportional to A ." [2]

2.1

- $\frac{dA}{dt} = kA$ with $k < 0$ (or $\frac{dA}{dt} = -kA$, $k > 0$)

2.2 A tank drains so that its volume V falls at a constant 3 litres per minute. Write the differential equation. [1]

2.2

- $\frac{dV}{dt} = -3$

2.3 Write a differential equation for: "the number of bacteria N grows at a rate proportional to N ." [1]

2.3

- $\frac{dN}{dt} = kN$

Exercise 3.1

3.1 "The rate of change of y is proportional to y " is written

[1]

☐

A $y = kt$

B $\frac{dy}{dt} = k$

C $\frac{dy}{dt} = ky$

D $\frac{dy}{dt} = k + y$

Exercise 3.1

3.1 "The rate of change of y is proportional to y " is written

[1]

☐

A $y = kt$

B $\frac{dy}{dt} = k$

C $\frac{dy}{dt} = ky$

D $\frac{dy}{dt} = k + y$

3.1 C

- proportional to y means $\frac{dy}{dt} = ky$

Exercise 3.2 (a)

3.2 A cup of coffee at temperature T cools in a room at 20°C . Its temperature falls at a rate proportional to how much hotter it is than the room.

(a) Write a differential equation for $\frac{dT}{dt}$.

[2]

Exercise 3.2 (a)

3.2 A cup of coffee at temperature T cools in a room at 20°C . Its temperature falls at a rate proportional to how much hotter it is than the room.

(a) Write a differential equation for $\frac{dT}{dt}$. [2]

(a)

- $\frac{dT}{dt} = k(T - 20)$

Exercise 3.2 (b)

3.2 A cup of coffee at temperature T cools in a room at 20°C . Its temperature falls at a rate proportional to how much hotter it is than the room.

(b) State the sign of the constant of proportionality, with a reason. [1]

Exercise 3.2 (b)

3.2 A cup of coffee at temperature T cools in a room at 20°C . Its temperature falls at a rate proportional to how much hotter it is than the room.

(b) State the sign of the constant of proportionality, with a reason. [1]

(b)

- $k < 0$ – the coffee is cooling, so its temperature decreases

Exercise 3.3

3.3 A savings account earns interest continuously at 4% per year, so the balance B grows at a rate of $0.04B$ per year, while \$2000 per year is also added. Write a differential equation for $\frac{dB}{dt}$. [2]

Exercise 3.3

3.3 A savings account earns interest continuously at 4% per year, so the balance B grows at a rate of $0.04B$ per year, while \$2000 per year is also added. Write a differential equation for $\frac{dB}{dt}$. [2]

3.3

- $\frac{dB}{dt} = 0.04B + 2000$