

Exercise walk-through

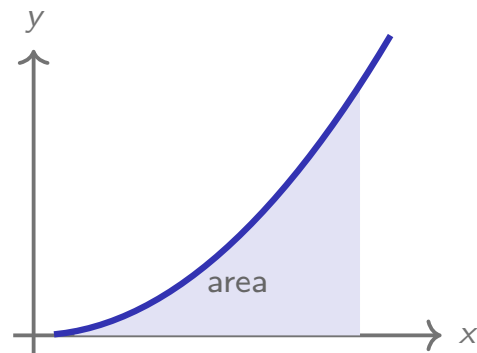
6.6

Applying Properties of Definite Integrals

AP Calculus BC

What we will cover

- 1 Accumulation of change
- 2 Riemann sums
- 3 The definite integral
- 4 The Fundamental Theorem
- 5 Antiderivatives
- 6 u -substitution
- 7 BC techniques



Accumulation functions and the FTC

A definite integral with a **variable upper limit** is an **accumulation function**. The

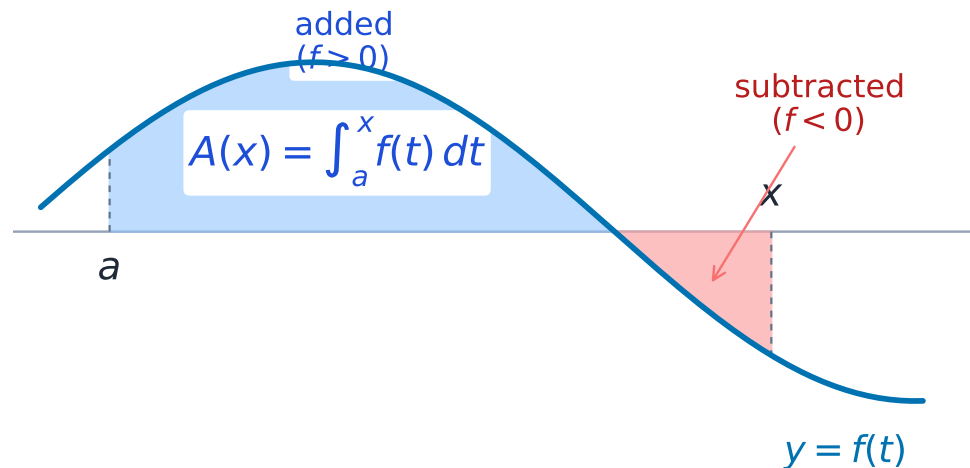
Fundamental Theorem

says differentiation undoes it:

$$\frac{d}{dx} \int_a^x f(t) dt = f(x).$$

So $g(x) = \int_a^x f$ has $g' = f$ and $g'' = f'$.

$$A'(x) = f(x) \quad (\text{Fundamental Theorem})$$



Exam tips

- Integration is antidifferentiation; use the **power rule** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ and don't forget the $+C$.
- The **Fundamental Theorem** links the two: $\int_a^b f'(x) dx = f(b) - f(a)$, and $\frac{d}{dx} \int_a^x f(t) dt = f(x)$.
- Approximate a definite integral with **Riemann sums** or the trapezoidal rule from a table of values.
- A definite integral is a signed area (below the axis counts negative); split at sign changes for total area.
- Use **u-substitution** and remember to change the limits (or back-substitute) accordingly.

Exercises 2.1·2.2

2.1 Find $\int_0^7 f \, dx$.

[1]

2.2 Find $\int_3^0 f \, dx$.

[1]

Exercises 2.1·2.2

2.1 Find $\int_0^7 f \, dx$.

[1]

2.1

- $\int_0^7 f = \int_0^3 f + \int_3^7 f = 6 + (-2) = 4$

2.2 Find $\int_3^0 f \, dx$.

[1]

2.2

- $\int_3^0 f = -\int_0^3 f = -6$

Exercises 2.3-3.1

2.3 Find $\int_0^3 (2f + g) dx$.

[2]

3.1 $\int_5 f(x) dx =$

[1]

☐**A** $f(5)$ **B** 1**C** 0**D** undefined

Exercises 2.3-3.1

2.3 Find $\int_0^3 (2f + g) dx$.

[2]

2.3

- $2(6) + 5 = 17$

3.1 $\int_5^5 f(x) dx =$

[1]

☐

A $f(5)$

B 1

C 0

D undefined

3.1 C

- a zero-width interval gives 0

Exercise 3.2 (a)

3.2 You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(a) Find $\int_4^{\sim} h \, dx$.

[2]

Exercise 3.2 (a)

3.2 You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(a) Find $\int_4^6 h \, dx$.

[2]

(a)

- $\int_4^6 h = \int_1^6 h - \int_1^4 h = 12 - 7 = 5$

Exercise 3.2 (b)

3.2 You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(b) Find $\int_6^4 h \, dx$.

[1]

Exercise 3.2 (b)

3.2 You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(b) Find $\int_6^1 h \, dx$.

[1]

(b)

- $\int_6^1 h = -12$

Exercise 3.3

3.3 Given $\int_0^2 p = 8$ and $\int_0^2 q = -3$, find $\int_0^2 (5p - 4q) dx$.

[2]

Exercise 3.3

3.3 Given $\int_0^2 p = 8$ and $\int_0^2 q = -3$, find $\int_0^2 (5p - 4q) dx$. [2]

3.3

- $5(8) - 4(-3) = 40 + 12 = 52$