

Exercise walk-through

6.4

The Fundamental Theorem of Calculus and Accumulation Functions

AP Calculus BC

Accumulation functions and the FTC

A definite integral with a **variable upper limit** is an **accumulation function**. The

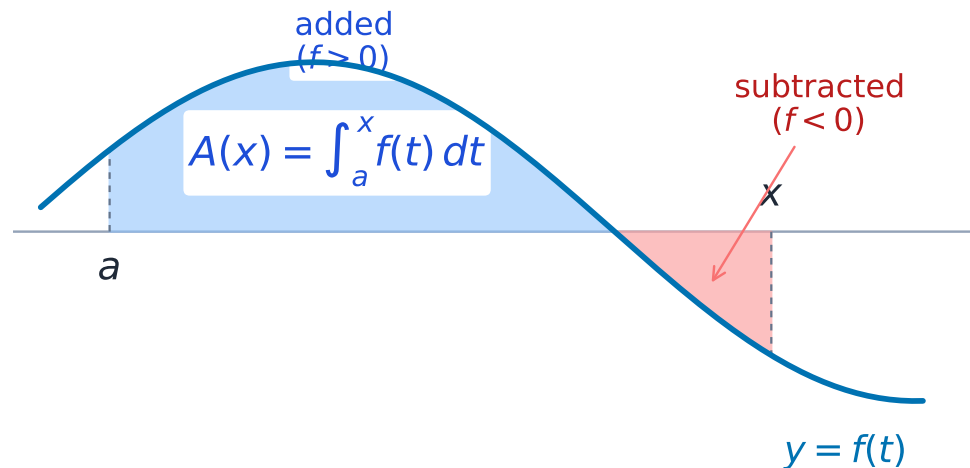
Fundamental Theorem

says differentiation undoes it:

$$\frac{d}{dx} \int_a^x f(t) dt = f(x).$$

So $g(x) = \int_a^x f$ has $g' = f$ and $g'' = f'$.

$$A'(x) = f(x) \quad (\text{Fundamental Theorem})$$



Behaviour of an accumulation function

Because $g'(x) = f(x)$, everything from Unit 5 applies – read it off the graph of f :

g **increasing** where $f > 0$,
decreasing where $f < 0$

g has a **local extremum**
where f **crosses zero**

g **concave up** where f is **increasing**

a **value** of g is the
signed area up to x

Accumulation functions, the FTC, and improper integrals

since $g'(x) = f(x)$

the graph of f tells you everything about g : g **increases** where $f > 0$, **decreases** where $f < 0$, and turns where f crosses zero

concavity of g

follows the *slope* of f

the FTC

its **evaluation** form computes a definite integral from an **antiderivative** : $\int_a^b f = F(b) - F(a)$

a trapezoidal sum

averages the left and right sums; it overestimates a concave-up function

an improper integral converges

if the limit exists: $\int_1^\infty \frac{1}{x^2} dx = \lim_{b \rightarrow \infty} \left[-\frac{1}{x}\right]_1^b = 1$

it diverges

if the limit does not – $\int_1^\infty \frac{1}{x} dx$ is the standard case

Write the limit explicitly before evaluating an improper integral. Skipping straight to the antiderivative loses the mark even when the answer is right.

Properties of definite integrals

$$\int_a^a f = 0$$

$$\int_b^a f = - \int_a^b f$$

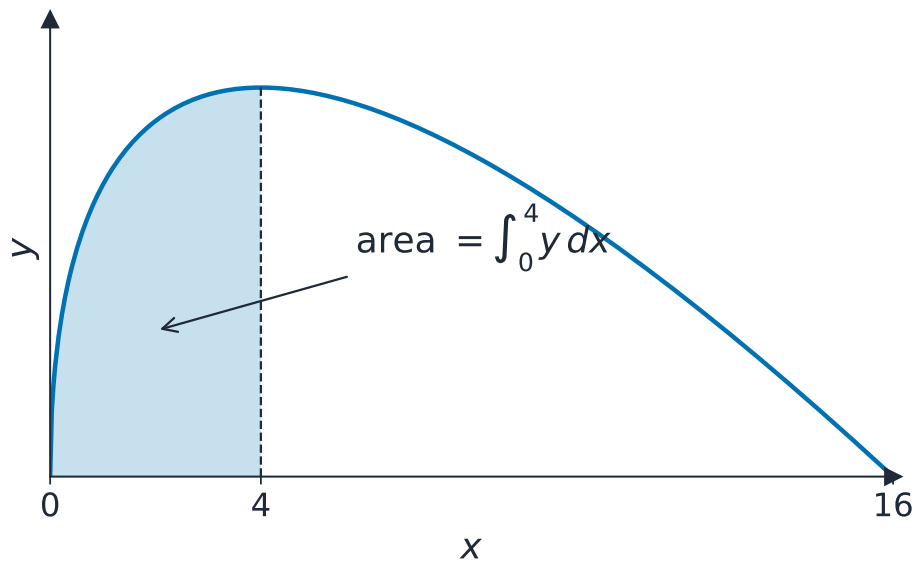
$$\int_a^b kf = k \int_a^b f$$

$$\int_a^b (f \pm g) = \int_a^b f \pm \int_a^b g$$

$$\int_a^c f + \int_c^b f = \int_a^b f$$

The last one – **splitting at an interior point** – lets you build a total from pieces read off a graph.

A definite integral is the signed area



A definite integral is the signed area between the curve and the x-axis

Exercises 2.1·2.2

2.1 If $g(x) = \int_0^x (t^3 + 1) dt$, find $g'(x)$. [1]

2.2 Find $\frac{d}{dx} \int_1^x \sqrt{t} dt$. [1]

Exercises 2.1·2.2

2.1 If $g(x) = \int_0^x (t^3 + 1) dt$, find $g'(x)$. [1]

2.1

- $g'(x) = x^3 + 1$
-

2.2 Find $\frac{d}{dx} \int_1^x \sqrt{t} dt$. [1]

2.2

- $\sqrt{x}$

Exercises 2.3-3.1

2.3 Find $\frac{d}{dx} \int_0^{x^3} \cos t \, dt$.

[2]

3.1 If $g(x) = \int_a^x f(t) \, dt$, then $g'(x) =$

[1]

☐**A** $f'(x)$ **B** $f(x)$ **C** $f(x) - f(a)$ **D** $F(x) - F(a)$

Exercises 2.3·3.1

2.3 Find $\frac{d}{dx} \int_0^{x^3} \cos t \, dt$.

[2]

2.3

- $\cos(x^3) \cdot 3x^2$

3.1 If $g(x) = \int_a^x f(t) \, dt$, then $g'(x) =$

[1]

☐

A $f'(x)$

B $f(x)$

C $f(x) - f(a)$

D $F(x) - F(a)$

3.1 B

- by the Fundamental Theorem, $g'(x) = f(x)$

Exercise 3.2 (a)

3.2 Let $g(x) = \int_2^x f(t) dt$, where f is continuous.

(a) Find $g(2)$.

[1]

Exercise 3.2 (a)

3.2 Let $g(x) = \int_2^x f(t) dt$, where f is continuous.

(a) Find $g(2)$.

[1]

(a)

- $g(2) = 0$ (zero-width interval)

Exercise 3.2 (b)

3.2 Let $g(x) = \int_2^x f(t) dt$, where f is continuous.

(b) Given $f(5) = 7$, find $g'(5)$.

[1]

Exercise 3.2 (b)

3.2 Let $g(x) = \int_2^x f(t) dt$, where f is continuous.

(b) Given $f(5) = 7$, find $g'(5)$.

[1]

(b)

- $g'(5) = f(5) = 7$

Exercise 3.2 (c)

3.2 Let $g(x) = \int_2^x f(t) dt$, where f is continuous.

(c) If $f(t) > 0$ for all $t > 2$, state whether g is increasing or decreasing for $x > 2$, with a reason. [2]

Exercise 3.2 (c)

3.2 Let $g(x) = \int_2^x f(t) dt$, where f is continuous.

(c) If $f(t) > 0$ for all $t > 2$, state whether g is increasing or decreasing for $x > 2$, with a reason. [2]

(c)

- Increasing – $g'(x) = f(x) > 0$ for $x > 2$

Exercise 3.3

3.3 Let $h(x) = \int_0^{2x} e^{-t^2} dt$. Find $h'(x)$.

[2]

Exercise 3.3

3.3 Let $h(x) = \int_0^{2x} e^{-t^2} dt$. Find $h'(x)$.

[2]

3.3

- $h'(x) = e^{-(2x)^2} \cdot 2 = 2e^{-4x^2}$