

Exercise walk-through

6.3

Riemann Sums, Summation Notation, and Definite Integral Notation

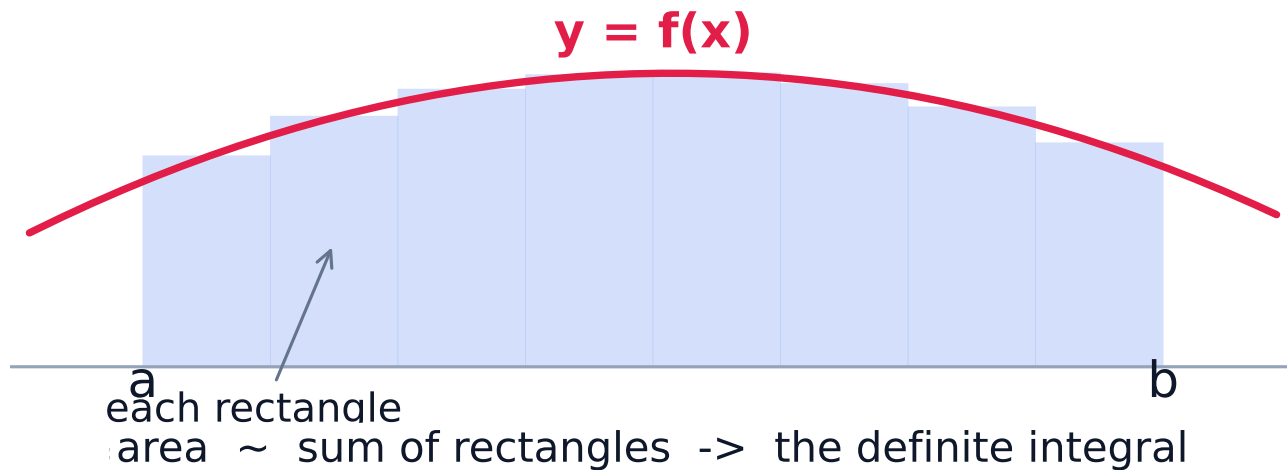
AP Calculus BC

Approximating area with a Riemann sum

Split the interval and add rectangle areas – a **Riemann sum**. Each rectangle is $f(x) \Delta x$.

As the strips narrow, the sum approaches the [definite integral](#).

Approximating area with a Riemann sum, in detail



$$\sum f(x_i^*) \Delta x \rightarrow \int_a^b f(x) dx \text{ as } n \rightarrow \infty$$

Split the interval and add rectangle areas – a **Riemann sum** .

The four standard estimates

Left sum – height from
each strip's **left** endpoint

Right sum – height
from the **right** endpoint

Midpoint sum – height
from the **midpoint**

Trapezoidal sum –
average the two endpoint heights

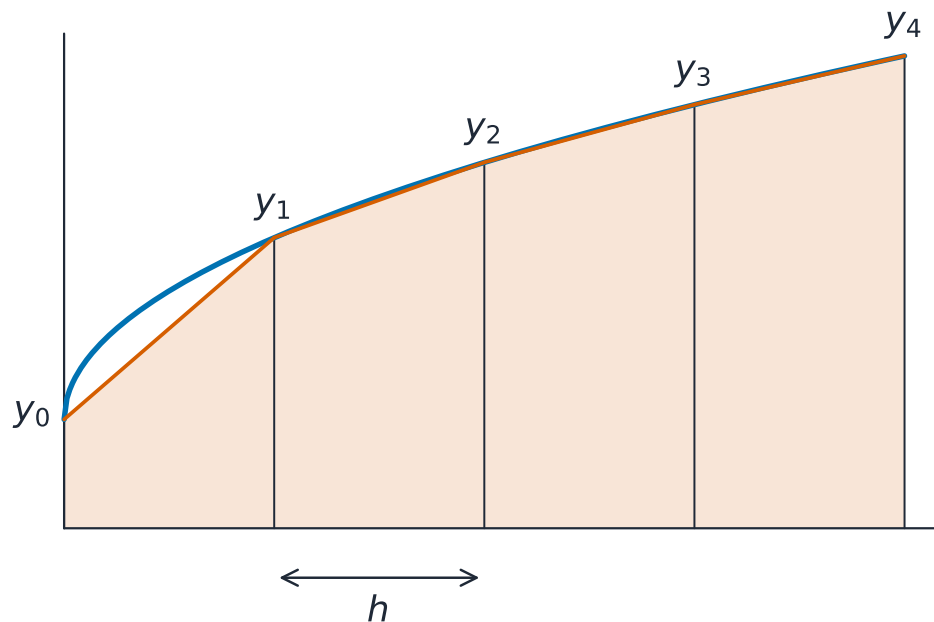
Strips may be **uniform** or **nonuniform** – read the widths from the table.

Over- or underestimate?

Judge from the **behaviour** of the function:

- **increasing** f : left sum **under**, right sum **over**
- **concave up**: trapezoidal **over**
- **concave down**: trapezoidal **under**

Over- or underestimate?, in detail



Exam parts ask you to state **which** and **why**.

Exercises 2.1·2.2

2.1 For $\int_2^{10} f \, dx$ estimated with $n = 4$ equal subintervals, state Δx . [1]

2.2 Write, in summation notation, a right Riemann sum for $\int_0^2 f \, dx$ with n equal subintervals. [2]

Exercises 2.1·2.2

2.1 For $\int_2^{10} f \, dx$ estimated with $n = 4$ equal subintervals, state Δx . [1]

2.1

- $\Delta x = \frac{10 - 2}{4} = 2$

2.2 Write, in summation notation, a right Riemann sum for $\int_0^2 f \, dx$ with n equal subintervals. [2]

2.2

- $\Delta x = \frac{2}{n}, x_k = \frac{2k}{n}; \text{sum} = \sum_{k=1}^n f\left(\frac{2k}{n}\right) \cdot \frac{2}{n}$

Exercises 2.3·3.1

2.3 In $\int_1^{} (3x + 2) dx$, name the integrand and the limits of integration. [2]

3.1 The definite integral $\int_a^b f dx$ is defined as [1]

☐

- A** an antiderivative of f
- B** the limit of Riemann sums as $n \rightarrow \infty$
- C** the slope of f
- D** the average of $f(a)$ and $f(b)$

Exercises 2.3-3.1

2.3 In $\int_1^5 (3x + 2) dx$, name the integrand and the limits of integration. [2]

2.3

- Integrand $3x + 2$; limits of integration 1 and 5

3.1 The definite integral $\int_a^b f dx$ is defined as [1]

☐

- A** an antiderivative of f
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3.1 B

- the definite integral is the limit of Riemann sums

Exercise 3.2 (a)

3.2 Consider $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{4k}{n}\right)^2 \cdot \frac{4}{n}$.

(a) State the width Δx and the interval $[a, b]$.

[2]

Exercise 3.2 (a)

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(a) State the width Δx and the interval $[a, b]$.

[2]

(a)

- $\Delta x = \frac{4}{n}$, so $b - a = 4$; the sample $2 + \frac{4k}{n}$ starts at 2, so $[a, b] = [2, 6]$

Exercise 3.2 (b)

3.2 Consider $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{4k}{n}\right)^2 \cdot \frac{4}{n}$.

(b) Write the limit as a definite integral.

[2]

Exercise 3.2 (b)

3.2 Consider $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{4k}{n}\right)^2 \cdot \frac{4}{n}$.

(b) Write the limit as a definite integral.

[2]

(b)

- $\int_2^6 x^2 dx$

Exercise 3.3

3.3 A definite integral $\int_0^6 v(t) dt$ evaluates to a negative number. Explain what a **negative** value tells you about the signed area, in one sentence. [1]

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3.3

- More signed area lies **below** the axis than above it over $[0, 6]$