

Exercise walk-through

6.14

Selecting Techniques for Antidifferentiation

AP Calculus BC

BC techniques for antidifferentiation

**Integration by
parts**

$$\int u \, dv = uv - \int v \, du$$

**Partial frac-
tions**

split a rational
into simple pieces

**Improper inte-
grals**

a limit at ∞ or
a discontinuity

Choose the technique by the form: a product $\rightarrow$ by parts, a rational $\rightarrow$ partial fractions, an infinite bound $\rightarrow$ an improper integral (evaluate with a limit).

Exam tips

- Integration is antidifferentiation; use the **power rule** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ and don't forget the $+C$.
- The **Fundamental Theorem** links the two: $\int_a^b f'(x) dx = f(b) - f(a)$, and $\frac{d}{dx} \int_a^x f(t) dt = f(x)$.
- Approximate a definite integral with **Riemann sums** or the trapezoidal rule from a table of values.
- A definite integral is a signed area (below the axis counts negative); split at sign changes for total area.
- Use **u-substitution** and remember to change the limits (or back-substitute) accordingly.

Exercises 2.1·2.3

$$2.1 \int \frac{1}{x^2 + 4} dx$$

[1]

$$2.2 \int 5x^4(x^5 + 2)^7 dx$$

[1]

$$2.3 \int \frac{x^2 + 1}{x} dx$$

[1]

Exercises 2.1-2.3

$$2.1 \int \frac{1}{x^2 + 4} dx \quad [1]$$

2.1

- Complete-the-square / arctangent form (basic arctangent rule)
-

$$2.2 \int 5x^4(x^5 + 2)^7 dx \quad [1]$$

2.2

- Substitution, $u = x^5 + 2$
-

$$2.3 \int \frac{x^2 + 1}{x} dx \quad [1]$$

2.3

- Basic – split into $x + \frac{1}{x}$ first

Exercises 3.1-3.2

3.1 Which integral is best done by **substitution**?

[1]

- ☐ A $\int (x^3 - 2) dx$
- ☐ B $\int \frac{1}{x^2 + 9} dx$
- ☐ C $\int \cos(x^2) \cdot 2x dx$
- ☐ D $\int \frac{x^2}{x + 1} dx$

3.2 Find $\int \frac{x^2 + 3}{x} dx$ (rewrite the fraction first).

[3]

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3.1 C

- $2x$ is the derivative of x^2 , so substitute $u = x^2$

3.2 Find $\int \frac{x^2 + 3}{x} dx$ (rewrite the fraction first).

[3]

3.2

- $\frac{x^2 + 3}{x} = x + \frac{3}{x}$; $\int \left(x + \frac{3}{x}\right) dx = \frac{x^2}{2} + 3 \ln |x| + C$

Exercise 3.3

3.3 Find $\int \frac{6x}{x^2 + 1} dx$.

[3]

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3.3

- $u = x^2 + 1$, $du = 2x dx$, so $6x dx = 3 du$; $\int \frac{3}{u} du = 3 \ln |u| = 3 \ln(x^2 + 1) + C$