

Exercise walk-through

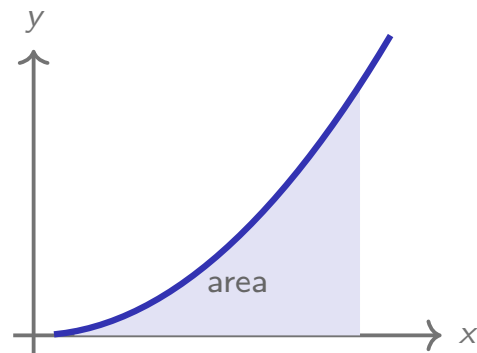
6.13

Evaluating Improper Integrals

AP Calculus BC

What we will cover

- 1 Accumulation of change
- 2 Riemann sums
- 3 The definite integral
- 4 The Fundamental Theorem
- 5 Antiderivatives
- 6 u -substitution
- 7 BC techniques



Accumulation functions, the FTC, and improper integrals

since $g'(x) = f(x)$

the graph of f tells you everything about g : g **increases** where $f > 0$, **decreases** where $f < 0$, and turns where f crosses zero

concavity of g

follows the *slope* of f

the FTC

its **evaluation** form computes a definite integral from an **antiderivative** : $\int_a^b f = F(b) - F(a)$

a trapezoidal sum

averages the left and right sums; it overestimates a concave-up function

an improper integral converges

if the limit exists: $\int_1^\infty \frac{1}{x^2} dx = \lim_{b \rightarrow \infty} \left[-\frac{1}{x}\right]_1^b = 1$

it diverges

if the limit does not – $\int_1^\infty \frac{1}{x} dx$ is the standard case

Write the limit explicitly before evaluating an improper integral. Skipping straight to the antiderivative loses the mark even when the answer is right.

BC techniques for antidifferentiation

**Integration by
parts**

$$\int u \, dv = uv - \int v \, du$$

**Partial frac-
tions**

split a rational
into simple pieces

**Improper inte-
grals**

a limit at ∞ or
a discontinuity

Choose the technique by the form: a product $\rightarrow$ by parts, a rational $\rightarrow$ partial fractions, an infinite bound $\rightarrow$ an improper integral (evaluate with a limit).

Exercises 2.1·2.2

2.1 Rewrite $\int_1^{\infty} \frac{1}{x^3} dx$ as a limit.

[1]

2.2 Evaluate $\int_1^{\infty} \frac{1}{x^3} dx$.

[3]

Exercises 2.1·2.2

2.1 Rewrite $\int_1^{\infty} \frac{1}{x^3} dx$ as a limit. [1]

2.1

- $\lim_{b \rightarrow \infty} \int_1^b x^{-3} dx$

2.2 Evaluate $\int_1^{\infty} \frac{1}{x^3} dx$. [3]

2.2

- $\left[-\frac{1}{2x^2} \right]_1^b = -\frac{1}{2b^2} + \frac{1}{2}; \text{ as } b \rightarrow \infty \text{ this } \rightarrow \frac{1}{2}$

Exercises 2.3·3.1

2.3 State whether $\int_1^{\infty} \frac{1}{x^{1/2}} dx$ converges or diverges (use the p -rule). [1]

3.1 $\int_1^{\infty} \frac{1}{x^p} dx$ converges when [1]

☐

A $p < 1$

B $p = 1$

C $p > 1$

D all p

Exercises 2.3-3.1

2.3 State whether $\int_1^{\infty} \frac{1}{x^{1/2}} dx$ converges or diverges (use the p -rule). [1]

2.3

- Diverges – $p = \frac{1}{2} \leq 1$

3.1 $\int_1^{\infty} \frac{1}{x^p} dx$ converges when [1]

- ☐ **A** $p < 1$
B $p = 1$
C $p > 1$
D all p

3.1 C

- converges for $p > 1$

Exercise 3.2 (a)

3.2 Evaluate $\int_0^{\infty} e^{-x} dx$.

(a) Write it as a limit.

[1]

Exercise 3.2 (a)

3.2 Evaluate $\int_0^{\infty} e^{-x} dx$.

(a) Write it as a limit.

[1]

(a)

- $\lim_{b \rightarrow \infty} \int_0^b e^{-x} dx$

Exercise 3.2 (b)

3.2 Evaluate $\int_0^{\infty} e^{-x} dx$.

(b) Evaluate.

[3]

Exercise 3.2 (b)

3.2 Evaluate $\int_0^{\infty} e^{-x} dx$.

(b) Evaluate.

[3]

(b)

- $\bullet \left[-e^{-x} \right]_0^b = -e^{-b} + 1; \rightarrow 1 \text{ as } b \rightarrow \infty$

Exercise 3.3

3.3 Determine whether $\int_2^{\infty} \frac{1}{x} dx$ converges or diverges, showing your reasoning with a limit. [3]

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3.3

- $\lim_{b \rightarrow \infty} [\ln x]_2^b = \lim_{b \rightarrow \infty} (\ln b - \ln 2); \ln b \rightarrow \infty$, so it **diverges**