

Exercise walk-through

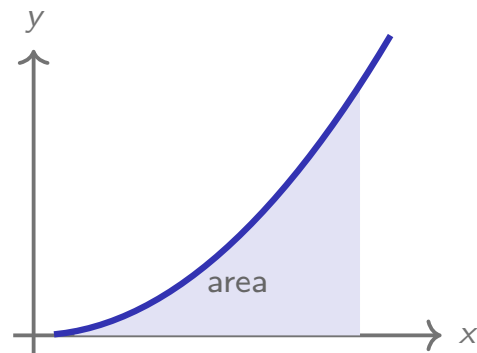
6.12

Integrating Using Linear Partial Fractions

AP Calculus BC

What we will cover

- 1 Accumulation of change
- 2 Riemann sums
- 3 The definite integral
- 4 The Fundamental Theorem
- 5 Antiderivatives
- 6 u -substitution
- 7 BC techniques



Integrating Using Linear Partial Fractions

Partial fractions split a rational function with a factorable denominator into a sum of simpler fractions:

$$\frac{1}{(x-a)(x-b)} = \frac{1}{a-b} \left(\frac{1}{x-a} - \frac{1}{x-b} \right).$$

This technique is essential for the logistic differential equation in the next unit.

BC techniques for antidifferentiation

**Integration by
parts**

$$\int u \, dv = uv - \int v \, du$$

**Partial frac-
tions**

split a rational
into simple pieces

**Improper inte-
grals**

a limit at ∞ or
a discontinuity

Choose the technique by the form: a product $\rightarrow$ by parts, a rational $\rightarrow$ partial fractions, an infinite bound $\rightarrow$ an improper integral (evaluate with a limit).

Exercises 2.1-2.3

2.1 Write the partial-fraction form of $\frac{1}{(x-3)(x+1)}$ (with unknowns A, B). [1]

2.2 Find A and B for $\frac{5}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$. [3]

2.3 Integrate $\int \left(\frac{2}{x-1} + \frac{3}{x+4} \right) dx$. [2]

Exercises 2.1-2.3

2.1 Write the partial-fraction form of $\frac{1}{(x-3)(x+1)}$ (with unknowns A, B). [1]

2.1

- $\frac{A}{x-3} + \frac{B}{x+1}$

2.2 Find A and B for $\frac{5}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$. [3]

2.2

- $5 = A(x+3) + B(x-2); x=2: 5 = 5A \Rightarrow A=1; x=-3: 5 = -5B \Rightarrow B=-1$

2.3 Integrate $\int \left(\frac{2}{x-1} + \frac{3}{x+4} \right) dx$. [2]

2.3

- $2 \ln|x-1| + 3 \ln|x+4| + C$

Exercise 3.1

3.1 After decomposing, $\int \frac{A}{x - a} dx =$

[1]

☐

A $A \ln |x - a| + C$

B $\frac{A}{(x - a)^2} + C$

C $A(x - a) + C$

D $\frac{A}{2}(x - a)^2 + C$

Exercise 3.1

3.1 After decomposing, $\int \frac{A}{x-a} dx =$

[1]

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A $A \ln |x - a| + C$

B $\frac{A}{(x-a)^2} + C$

C $A(x-a) + C$

D $\frac{A}{2}(x-a)^2 + C$

3.1 A

- $\int \frac{A}{x-a} dx = A \ln |x - a| + C$

Exercise 3.2 (a)

3.2 Decompose and integrate $\int \frac{1}{(x-1)(x+1)} dx$.

(a) Find the constants.

[2]

Exercise 3.2 (a)

3.2 Decompose and integrate $\int \frac{1}{(x-1)(x+1)} dx$.

(a) Find the constants.

[2]

(a)

- $1 = A(x+1) + B(x-1); x = 1: A = \frac{1}{2}; x = -1: B = -\frac{1}{2}$

Exercise 3.2 (b)

3.2 Decompose and integrate $\int \frac{1}{(x-1)(x+1)} dx$.

(b) Complete the integration.

[2]

Exercise 3.2 (b)

3.2 Decompose and integrate $\int \frac{1}{(x-1)(x+1)} dx$.

(b) Complete the integration.

[2]

(b)

- $\frac{1}{2} \ln |x-1| - \frac{1}{2} \ln |x+1| + C$

Exercise 3.3

3.3 Find $\int \frac{x+7}{(x-1)(x+3)} dx$.

[5]

Exercise 3.3

3.3 Find $\int \frac{x+7}{(x-1)(x+3)} dx$. [5]

3.3

- $x+7 = A(x+3) + B(x-1)$; $x=1$: $8 = 4A \Rightarrow A=2$; $x=-3$: $4 = -4B \Rightarrow B=-1$;
 $\int \left(\frac{2}{x-1} - \frac{1}{x+3} \right) dx = 2 \ln |x-1| - \ln |x+3| + C$