

Exercise walk-through

6.11

Integrating Using Integration by Parts

AP Calculus BC

Integrating Using Integration by Parts

Integration by parts

reverses the product rule:

$$\int u \, dv = uv - \int v \, du.$$

Choose u to simplify when differentiated a.

It handles products like $\int x e^x \, dx$ and $\int x \ln x \, dx$, sometimes applied twice.

BC techniques for antidifferentiation

**Integration by
parts**

$$\int u \, dv = uv - \int v \, du$$

**Partial frac-
tions**

split a rational
into simple pieces

**Improper inte-
grals**

a limit at ∞ or
a discontinuity

Choose the technique by the form: a product $\rightarrow$ by parts, a rational $\rightarrow$ partial fractions, an infinite bound $\rightarrow$ an improper integral (evaluate with a limit).

Exam tips

- Integration is antidifferentiation; use the **power rule** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ and don't forget the $+C$.
- The **Fundamental Theorem** links the two: $\int_a^b f'(x) dx = f(b) - f(a)$, and $\frac{d}{dx} \int_a^x f(t) dt = f(x)$.
- Approximate a definite integral with **Riemann sums** or the trapezoidal rule from a table of values.
- A definite integral is a signed area (below the axis counts negative); split at sign changes for total area.
- Use **u-substitution** and remember to change the limits (or back-substitute) accordingly.

Exercises 2.1-2.3

2.1 For $\int x \cos x \, dx$, state a good choice of u and dv . [2]

2.2 Find $\int x \cos x \, dx$. [3]

2.3 Find $\int x e^{2x} \, dx$. [3]

Exercises 2.1·2.3

2.1 For $\int x \cos x \, dx$, state a good choice of u and dv . [2]

2.1

- $u = x$ (algebraic), $dv = \cos x \, dx$
-

2.2 Find $\int x \cos x \, dx$. [3]

2.2

- $du = dx$, $v = \sin x$; $\int x \cos x \, dx = x \sin x - \int \sin x \, dx = x \sin x + \cos x + C$
-

2.3 Find $\int x e^{2x} \, dx$. [3]

2.3

- $u = x$, $dv = e^{2x} \, dx \Rightarrow v = \frac{1}{2}e^{2x}$; $\frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x} \, dx = \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x} + C$

Exercises 3.1·3.2

3.1 In $\int x \sin x \, dx$, the best choice by LIATE is [1]

☐

A $u = \sin x, dv = x \, dx$

B $u = x, dv = \sin x \, dx$

C $u = x \sin x, dv = dx$

D $u = 1, dv = x \sin x \, dx$

3.2 Find $\int \ln x \, dx$. [3]

Exercises 3.1-3.2

3.1 In $\int x \sin x \, dx$, the best choice by LIATE is [1]

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A $u = \sin x$, $dv = x \, dx$

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C $u = x \sin x$, $dv = dx$

D $u = 1$, $dv = x \sin x \, dx$

3.1 B

- x is algebraic (A), $\sin x$ is trig (T); A comes before T, so $u = x$

3.2 Find $\int \ln x \, dx$. [3]

3.2

- $u = \ln x$, $dv = dx$, $v = x$; $x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - x + C$

Exercise 3.3

3.3 Evaluate $\int_0^1 xe^x dx$.

[4]

Exercise 3.3

3.3 Evaluate $\int_0^1 xe^x dx$.

[4]

3.3

- $u = x, dv = e^x dx; [xe^x]_0^1 - \int_0^1 e^x dx = (e - 0) - [e^x]_0^1 = e - (e - 1) = 1$