

Exercise walk-through

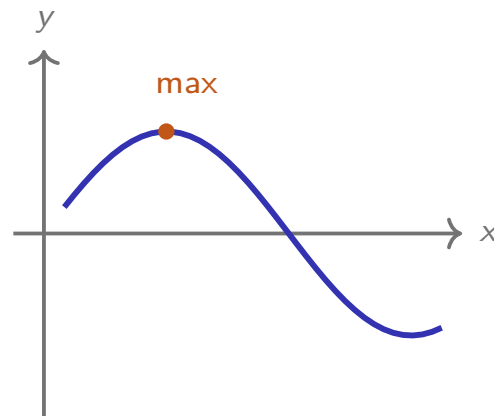
5.8

Sketching Graphs of Functions and Their Derivatives

AP Calculus BC

What we will cover

- 1 The Mean Value Theorem
- 2 Critical points & extrema
- 3 The first derivative test
- 4 Concavity & inflection
- 5 Connecting f , f' and f''
- 6 Optimization



Exam skill – justifying the MVT

The **Mean Value Theorem** is an **existence** theorem, so the marks are in the justification:

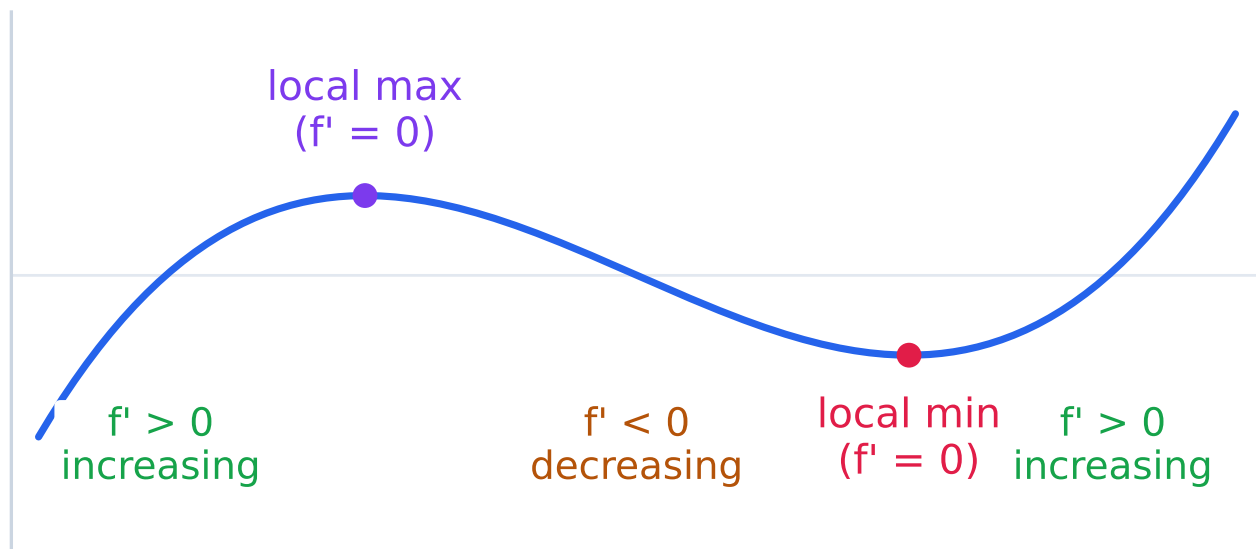
- 1 state f is continuous on $[a, b]$ and differentiable on (a, b)
- 2 compute $\frac{f(b) - f(a)}{b - a}$
- 3 conclude “by the MVT there is a c in (a, b) with $f'(c)$ equal to that value”

Both hypotheses must be named – dropping one loses the mark even with the right c .

Verify the MVT for $f(x) = x^2$ on $[1, 3]$

A polynomial, so continuous and differentiable everywhere. Average rate $\frac{9 - 1}{2} = 4$; solve $f'(c) = 2c = 4$ to get $c = 2$, which lies in $(1, 3)$.

Worked example – classify the critical points, in detail



$f' > 0 \Rightarrow$ increasing · $f' < 0 \Rightarrow$ decreasing · $f' = 0$ candidates for extrema

$$f(x) = x^3 - 3x^2 \quad f'(x) = 3x^2 - 6x = 3x(x - 2), \text{ zero at } x = 0, 2.$$

Exercises 2.1-2.3

2.1 State the value of f' at a local extremum of f . [1]

2.2 State the sign of f' where f is increasing. [1]

2.3 State what f' does where f is concave up. [2]

Exercises 2.1-2.3

2.1 State the value of f' at a local extremum of f . [1]

2.1

- $f' = 0$
-

2.2 State the sign of f' where f is increasing. [1]

2.2

- positive ($f' > 0$)
-

2.3 State what f' does where f is concave up. [2]

2.3

- f' is increasing

Exercises 3.1-3.2

3.1 At a local max of f , f' is

[1]

☐

A positive

B zero

C negative always

D undefined

3.2 Where f is decreasing, its derivative f' is

[1]

☐

A positive

B negative

C zero

D increasing

Exercises 3.1-3.2

3.1 At a local max of f , f' is

[1]

☐

A positive

B zero

C negative always

D undefined

3.1 B

- zero.

3.2 Where f is decreasing, its derivative f' is

[1]

☐

A positive

B negative

C zero

D increasing

3.2 B

- negative.

Exercise 3.3 (a)

3.3 f has a local minimum at $x = 2$.

(a) State $f'(2)$.

[1]

Exercise 3.3 (a)

3.3 f has a local minimum at $x = 2$.

(a) State $f'(2)$.

[1]

(a)

- 0

Exercise 3.3 (b)

3.3 f has a local minimum at $x = 2$.

(b) State the sign of f' just left of 2.

[1]

Exercise 3.3 (b)

3.3 f has a local minimum at $x = 2$.

(b) State the sign of f' just left of 2.

[1]

(b)

- negative

Exercise 3.3 (c)

3.3 f has a local minimum at $x = 2$.

(c) State the sign just right of 2.

[1]

Exercise 3.3 (c)

3.3 f has a local minimum at $x = 2$.

(c) State the sign just right of 2.

[1]

(c)

- positive