

Exercise walk-through

5.6

Determining Concavity of Functions over Their Domains

AP Calculus BC

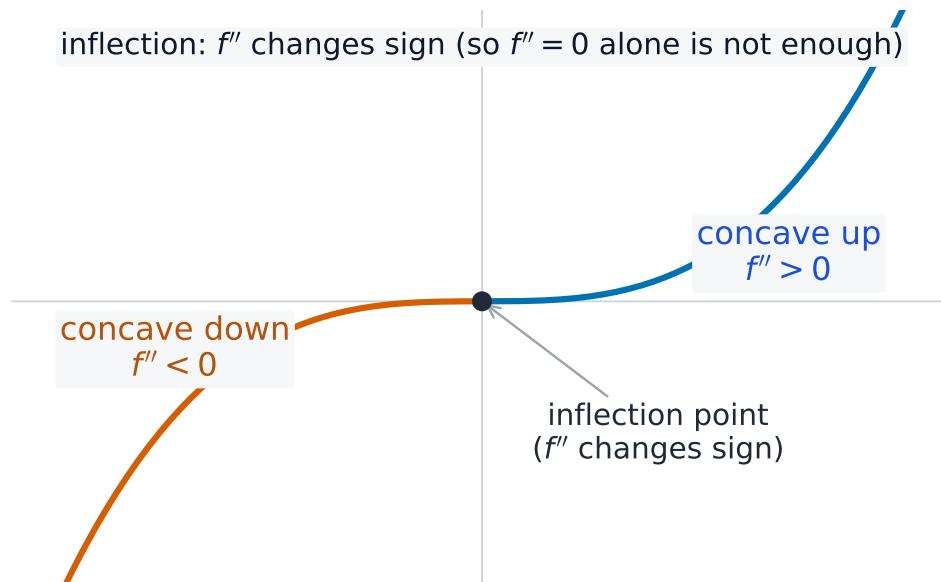
Concavity and points of inflection

The **second derivative** describes bending:

- $f'' > 0$: **concave up** (f' increasing)
- $f'' < 0$: **concave down** (f' decreasing)

A **point of inflection** is where f'' **changes sign** – not merely where $f'' = 0$.

Concavity and points of inflection, in detail



Concavity comes from the sign of the second derivative; it flips at an inflection point

The second derivative test

An alternative at a critical point c with $f'(c) = 0$:

$$f''(c) > 0 \Rightarrow \text{local minimum}$$

$$f''(c) < 0 \Rightarrow \text{local maximum}$$

$$f''(c) = 0 \Rightarrow \text{inconclusive}$$

If $f''(c) = 0$, fall back on the **first derivative test**.

Special case: if a continuous function has only **one** critical point on an interval and it is a local extremum, it is the **absolute** extremum there.

Connecting f , f' and f''

A very common setup: you are given the graph of f' and asked about f .

f **increasing** $\Leftrightarrow f'$ **above the axis**

f has a **local max** $\Leftrightarrow$
 f' crosses $+$ $\rightarrow$ $-$

f **concave up** $\Leftrightarrow f''$ **increasing**

f has an **inflection** $\Leftrightarrow$
 f' has a local extremum

Answer questions about f using the **height** and **slope** of the f' graph.

Exercises 2.1-2.3

2.1 State where f is concave up.

[1]

2.2 For $f(x) = x^3$, find f'' and where it is concave up.

[2]

2.3 State what an inflection point is.

[1]

Exercises 2.1·2.3

2.1 State where f is concave up. [1]

2.1

- where $f'' > 0$
-

2.2 For $f(x) = x^3$, find f'' and where it is concave up. [2]

2.2

- $f''(x) = 6x$; concave up for $x > 0$
-

2.3 State what an inflection point is. [1]

2.3

- a point where concavity changes (from up to down or vice versa)

Exercises 3.1-3.2

3.1 f is concave up where

[1]

☐

A $f' > 0$

B $f'' > 0$

C $f'' < 0$

D $f = 0$

☐

3.2 An inflection point is where

[1]

A $f' = 0$

B concavity changes

C $f = 0$

D f'' is constant

Exercises 3.1-3.2

3.1 f is concave up where

[1]

- ☐ A $f' > 0$
☐ B $f'' > 0$
☐ C $f'' < 0$
☐ D $f = 0$

3.1 B

- $f'' > 0$.

3.2 An inflection point is where

[1]

- ☐ A $f' = 0$
☐ B concavity changes
☐ C $f = 0$
☐ D f'' is constant

3.2 B

- concavity changes.

Exercise 3.3 (a)

3.3 $f(x) = x^3$.

(a) Find $f''(x)$.

[1]

Exercise 3.3 (a)

3.3 $f(x) = x^3$.

(a) Find $f''(x)$.

[1]

(a)

- $6x$

Exercise 3.3 (b)

3.3 $f(x) = x^3$.

(b) State where $f'' > 0$.

[1]

Exercise 3.3 (b)

3.3 $f(x) = x^3$.

(b) State where $f'' > 0$.

[1]

(b)

- $x > 0$

Exercise 3.3 (c)

3.3 $f(x) = x^3$.

(c) State the inflection point.

[1]

Exercise 3.3 (c)

3.3 $f(x) = x^3$.

(c) State the inflection point.

[1]

(c)

- $x = 0$