

Exercise walk-through

5.3

Determining Intervals on Which a Function Is Increasing or Decreasing

AP Calculus BC

Exam skill – justifying the MVT

The **Mean Value Theorem** is an **existence** theorem, so the marks are in the justification:

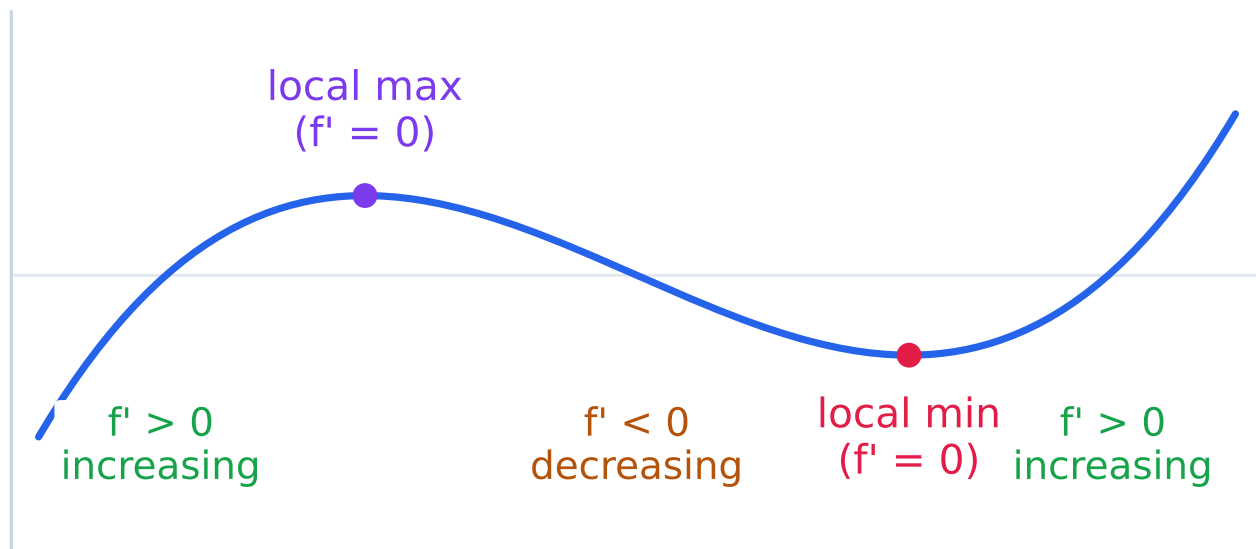
- 1 state f is continuous on $[a, b]$ **and** differentiable on (a, b)
- 2 compute $\frac{f(b) - f(a)}{b - a}$
- 3 conclude “by the MVT there is a c in (a, b) with $f'(c)$ equal to that value”

Both hypotheses must be named – dropping one loses the mark even with the right c .

Verify the MVT for $f(x) = x^2$ on $[1, 3]$

A polynomial, so continuous and differentiable everywhere. Average rate $\frac{9 - 1}{2} = 4$; solve $f'(c) = 2c = 4$ to get $c = 2$, which lies in $(1, 3)$.

Worked example – classify the critical points, in detail



$f' > 0 \Rightarrow$ increasing $\cdot f' < 0 \Rightarrow$ decreasing $\cdot f' = 0$ candidates for extrema

$$f(x) = x^3 - 3x^2 \quad f'(x) = 3x^2 - 6x = 3x(x - 2), \text{ zero at } x = 0, 2.$$

Connecting f , f' and f''

A very common setup: you are given the graph of f' and asked about f .

f **increasing** $\Leftrightarrow f'$ **above the axis**

f has a **local max** $\Leftrightarrow$
 f' crosses $+$ $\rightarrow$ $-$

f **concave up** $\Leftrightarrow f''$ **increasing**

f has an **inflection** $\Leftrightarrow$
 f'' has a local extremum

Answer questions about f using the **height** and **slope** of the f' graph.

Exercises 2.1-2.3

2.1 State where f is increasing.

[1]

2.2 For $f'(x) = 2x - 4$, find where f is increasing.

[2]

2.3 For the same f' , state where f is decreasing.

[1]

Exercises 2.1·2.3

2.1 State where f is increasing. [1]

2.1

- where $f' > 0$
-

2.2 For $f'(x) = 2x - 4$, find where f is increasing. [2]

2.2

- $2x - 4 > 0 \Rightarrow x > 2$
-

2.3 For the same f' , state where f is decreasing. [1]

2.3

- $x < 2$

Exercises 3.1-3.2

3.1 f is increasing where

[1]

☐

A $f' < 0$

B $f' > 0$

C $f'' > 0$

D $f = 0$

☐

3.2 If $f'(x) = 2x - 4$, then f is decreasing for

[1]

A $x > 2$

B $x < 2$

C all x

D $x = 2$

Exercises 3.1·3.2

3.1 f is increasing where

[1]

☐

A $f' < 0$

B $f' > 0$

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3.1 B

- $f' > 0$.

3.2 If $f'(x) = 2x - 4$, then f is decreasing for

[1]

☐

A $x > 2$

B $x < 2$

C all x

D $x = 2$

3.2 B

- $x < 2$.

Exercise 3.3 (a)

3.3 $f'(x) = 3x^2 - 3$.

(a) Solve $f' = 0$.

[1]

Exercise 3.3 (a)

3.3 $f'(x) = 3x^2 - 3$.

(a) Solve $f' = 0$.

[1]

(a)

- $x = \pm 1$

Exercise 3.3 (b)

3.3 $f'(x) = 3x^2 - 3$.

(b) Test a point in $(-1, 1)$.

[1]

Exercise 3.3 (b)

3.3 $f'(x) = 3x^2 - 3$.

(b) Test a point in $(-1, 1)$.

[1]

(b)

- $f'(0) = -3 < 0$

Exercise 3.3 (c)

3.3 $f'(x) = 3x^2 - 3$.

(c) State where f is decreasing.

[1]

Exercise 3.3 (c)

3.3 $f'(x) = 3x^2 - 3$.

(c) State where f is decreasing.

[1]

(c)

- $(-1, 1)$