

Exercise walk-through

5.12

Exploring Behaviors of Implicit Relations

AP Calculus BC

You must be able to

- find where an implicit curve has a **horizontal tangent** ($\frac{dy}{dx} = 0$) or a **vertical tangent** ($\frac{dy}{dx}$ undefined)
- decide **concavity** from the sign of $\frac{d^2y}{dx^2}$ on the curve
- evaluate these using a point that lies **on** the curve

1

The method

Method — Horizontal tangents

For $x^2 + y^2 = 25$, implicit differentiation gives $2x + 2y \frac{dy}{dx} = 0$, so

$$\frac{dy}{dx} = -\frac{x}{y}.$$

A **horizontal** tangent needs $\frac{dy}{dx} = 0$, i.e. the numerator $x = 0$ (with $y \neq 0$): the points $(0, 5)$ and $(0, -5)$.

Exercises 2.1-2.3

2.1 For $x^2 + y^2 = 25$, state $\frac{dy}{dx}$. [1]

2.2 Find the point(s) on $x^2 + y^2 = 25$ where the tangent is **horizontal**. [2]

2.3 A curve satisfies $\frac{dy}{dx} = \frac{2x}{y}$. State where its tangent line is **vertical**. [1]

Exercises 2.1-2.3

2.1 For $x^2 + y^2 = 25$, state $\frac{dy}{dx}$. [1]

2.1

- $\frac{dy}{dx} = -\frac{x}{y}$

2.2 Find the point(s) on $x^2 + y^2 = 25$ where the tangent is **horizontal**. [2]

2.2

- Set numerator $x = 0$; points $(0, 5)$ and $(0, -5)$

2.3 A curve satisfies $\frac{dy}{dx} = \frac{2x}{y}$. State where its tangent line is **vertical**. [1]

2.3

- Vertical where denominator $y = 0$, i.e. at $(\pm 5, 0)$ – where the curve meets the x -axis

Exercise 3.1

3.1 A curve has $\frac{dy}{dx} = \frac{x-1}{y}$. A horizontal tangent occurs where

[1]

☐**A** $y = 0$ **B** $x = 0$ **C** $x = 1$ **D** $x = y$

Exercise 3.1

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[1]

☐**A** $y = 0$ **B** $x = 0$ **C** $x = 1$ **D** $x = y$

3.1 C

- a horizontal tangent needs the numerator zero, $x - 1 = 0$, so $x = 1$

Exercise 3.2 (a)

3.2 The curve $x^2 + xy + y^2 = 7$ passes through $(1, 2)$. Its derivative is $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$.

(a) Find the slope of the tangent at $(1, 2)$.

[2]

Exercise 3.2 (a)

3.2 The curve $x^2 + xy + y^2 = 7$ passes through $(1, 2)$. Its derivative is $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$.

(a) Find the slope of the tangent at $(1, 2)$. [2]

(a)

- $\frac{dy}{dx} = -\frac{2(1) + 2}{1 + 2(2)} = -\frac{4}{5}$

Exercise 3.2 (b)

3.2 The curve $x^2 + xy + y^2 = 7$ passes through $(1, 2)$. Its derivative is $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$.

(b) Determine whether the tangent at $(1, 2)$ is horizontal, vertical, or neither. [1]

Exercise 3.2 (b)

3.2 The curve $x^2 + xy + y^2 = 7$ passes through $(1, 2)$. Its derivative is $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$.

(b) Determine whether the tangent at $(1, 2)$ is horizontal, vertical, or neither. [1]

(b)

- Neither – the slope $-\frac{4}{5}$ is finite and non-zero

Exercise 3.3

3.3 For $x^2 + y^2 = 25$ it can be shown that $\frac{d^2y}{dx^2} = -\frac{25}{y^3}$. State, with a reason, whether the curve is concave up or concave down at the point $(3, 4)$. [2]

$$x^2 + y^2 = 25$$

$$(3, 4)$$

$$\frac{d^2y}{dx^2} = -\frac{25}{y^3}$$

Exercise 3.3

3.3 For $x^2 + y^2 = 25$ it can be shown that $\frac{d^2y}{dx^2} = -\frac{25}{y^3}$. State, with a reason, whether the curve is concave up or concave down at the point $(3, 4)$. [2]

$$x^2 + y^2 = 25$$

$$(3, 4)$$

$$\frac{d^2y}{dx^2} = -\frac{25}{y^3}$$

3.3

- $\frac{d^2y}{dx^2} = -\frac{25}{4^3} = -\frac{25}{64} < 0$; negative, so the curve is **concave down** at $(3, 4)$