Exercise walk-through

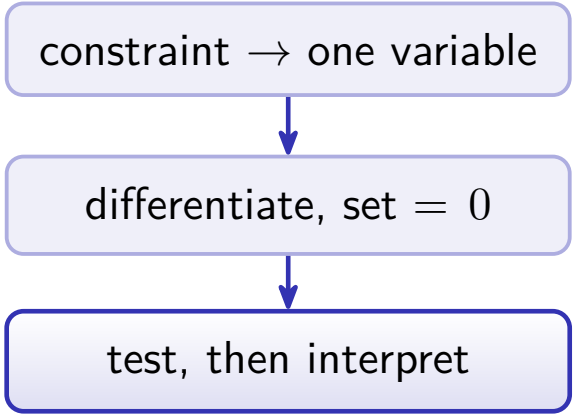
5.11
Solving Optimization Problems

AP Calculus BC

Solving an optimization problem

Optimization is the same machinery aimed at a real goal:

1. write the quantity as a function of one variable (use the constraint to eliminate the rest)
2. state the allowed interval
3. find and test the critical points
4. answer with units and interpretation



Worked example – the farmer's fence

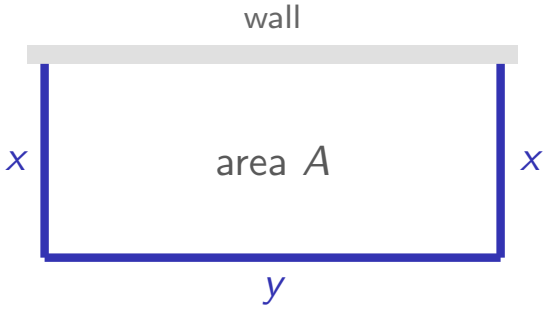
$100\ \mathrm{m}$ of fence, three sides (a wall is the fourth). Maximize the area.

Ends x, side y: constraint $2x + y = 100$, so $y = 100 - 2x$.

$$A(x) = x(100 - 2x) = 100x - 2x^2$$

$$A'(x) = 100 - 4x = 0 \;\Rightarrow\; x = 25$$

$A'' = -4 < 0$, a maximum: $y = 50$ and $A = 1250\ \mathrm{m}^2$.



Behaviour of implicit relations

It all extends to implicitly defined relations:

- horizontal tangent where $\frac{dy}{dx} = 0$
- vertical tangent where $\frac{dy}{dx}$ is undefined

$\frac{dy}{dx}$ usually involves both x and y – substitute it back in when finding $\frac{d^2y}{dx^2}$, then read concavity from the sign.

Exercises 2.1·2.3

2.1 For $A(y) = 40y - 2y^2$, find $A'(y)$ and solve $A'(y) = 0$. [2]

2.2 The area function is $A = 12x - x^2$. Find the width x that maximizes the area, and **justify** it is a maximum. [3]

2.3 A function $C(x) = x^2 + \dfrac{16}{x}$ models a cost for $x > 0$. Find the x that minimizes C. [3]

Exercises 2.1·2.3

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2.1

- $A'(y) = 40 - 4y$; $40 - 4y = 0 \Rightarrow y = 10$

2.2 The area function is $A = 12x - x^2$. Find the width x that maximizes the area, and **justify** it is a maximum. [3]

2.2

- $A'(x) = 12 - 2x = 0 \Rightarrow x = 6$; $A''(x) = -2 < 0$, so $x = 6$ is a maximum

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2.3 A function $C(x) = x^2 + \dfrac{16}{x}$ models a cost for $x > 0$. Find the x that minimizes C. [3]

2.3

- $C'(x) = 2x - \dfrac{16}{x^2} = 0$; $2x^3 = 16 \Rightarrow x^3 = 8 \Rightarrow x = 2$

Exercise 3.1

3.1 A critical point of f has $f'(c) = 0$ and $f''(c) < 0$. The point is a [1]

- **A** local minimum
- **B** local maximum
- **C** point of inflection
- **D** endpoint

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3.1 B

- $f'

(c)

- $= 0 with f''

(c)

- < 0 is a local maximum

Exercise 3.2 (a)

3.2 A rectangular field of area $A = x(200 - x)$ m^2 is enclosed, where $0 < x < 200$.

(a) Find the value of x that maximizes A. [2]

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(a)

- $A = 200x - x^2$, $A'(x) = 200 - 2x = 0 \Rightarrow x = 100$

Exercise 3.2 (b)

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(b) Justify that your value gives a maximum. [1]

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(c) State the maximum area. [1]

(c)

- $A = 100(100) = 10\,000$ m^2

Exercise 3.3 (a)

3.3 An open box is made from card, with volume $V(x) = 150x - \frac{1}{4}x^3$ for $0 < x < \sqrt{600}$.

(a) Show that $V'(x) = 150 - \frac{3}{4}x^2$. [1]

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Exercise 3.3 (b)

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(b)

- $150 - \frac{3}{4}x^2 = 0 \Rightarrow x^2 = 200 \Rightarrow x = \sqrt{200} \approx 14.1$; $V''(x) = -\frac{3}{2}x < 0$ for $x > 0$, so it is a maximum