Exercise walk-through

4.7 Using L'Hospital's Rule for Determining Limits of Indeterminate Forms

AP Calculus BC

L'Hospital's rule

If substitution gives the **indeterminate form** $\frac{0}{0}$ or $\frac{\infty}{\infty}$:

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}.$$

> Differentiate top and bottom separately – this is not the quotient rule.

Evaluate $\lim_{x \to 0} \frac{\sin x}{x}$

Substituting gives $\frac{0}{0}$, so differentiate top and bottom:

$$\lim_{x \to 0} \frac{\cos x}{1} = \cos 0 = 1$$

– the famous limit from Unit 1.

Check the form first – applying the rule to anything else is a mistake.

Introduction to Related Rates

A rising balloon: related rates link how fast radius, volume and height change together

Exercises 2.1·2.3

2.1 State the forms L'Hospital's Rule applies to. [1]

2.2 Apply it to $\lim_{x\to 0} \dfrac{\sin x}{x}$. [2]

2.3 State what you do to the numerator and denominator. [1]

Exercises 2.1·2.3

2.1 State the forms L'Hospital's Rule applies to. [1]

2.1

- $\frac{0}{0}$ and $\frac{\infty}{\infty}$

2.2 Apply it to $\lim_{x\to 0} \dfrac{\sin x}{x}$. [2]

2.2

- $\lim_{x\to 0} \dfrac{\cos x}{1} = 1$

2.3 State what you do to the numerator and denominator. [1]

2.3

- differentiate each separately, then take the limit of the new quotient

Exercise 3.1

3.1 L'Hospital's Rule applies to [1]

A any limit
B $\frac{0}{0}$ or $\frac{\infty}{\infty}$
C $\frac{1}{0}$
D $\frac{5}{2}$

Exercise 3.1

3.1 L'Hospital's Rule applies to [1]

A any limit
B $\frac{0}{0}$ or $\frac{\infty}{\infty}$
C $\frac{1}{0}$
D $\frac{5}{2}$

3.1 B

- $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Exercise 3.2

3.2 L'Hospital's Rule replaces $\dfrac{f}{g}$ with [1]

A $f \cdot g$

B $\dfrac{f'}{g'}$

C $\dfrac{f'}{g}$

D $\dfrac{f}{g'}$

Exercise 3.2

3.2 L'Hospital's Rule replaces $\dfrac{f}{g}$ with [1]

- **A** $f \cdot g$
- **B** $\dfrac{f'}{g'}$
- **C** $\dfrac{f'}{g}$
- **D** $\dfrac{f}{g'}$

3.2 B

- $\dfrac{f'}{g'}$.

Exercise 3.3 (a)

3.3 $\lim_{x \to 0} \dfrac{e^x - 1}{x}$.

(a) Check the form. [1]

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(a) Check the form. [1]

(a)

- $\frac{0}{0}$

Exercise 3.3 (b)

3.3 $\lim_{x \to 0} \dfrac{e^x - 1}{x}$.

(b) Differentiate top and bottom. [1]

Exercise 3.3 (b)

3.3 $\lim_{x \to 0} \dfrac{e^x - 1}{x}$.

(b) Differentiate top and bottom. [1]

(b)

- $\dfrac{e^x}{1}$

Exercise 3.3 (c)

3.3 $\lim_{x \to 0} \dfrac{e^x - 1}{x}$.

(c) Evaluate. [1]

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3.3 $\lim_{x \to 0} \dfrac{e^x - 1}{x}$.

(c) Evaluate. [1]

(c)

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