Exercise walk-through

4.4
Introduction to Related Rates

AP Calculus BC

Related rates – the five-step template

In a **related rates** problem, quantities change together in time. The engine is the chain rule: differentiate with respect to t.

1. name the variables; list given & wanted rates
2. write an equation relating them
3. differentiate both sides in t
4. substitute the values last
5. state units and the correct sign

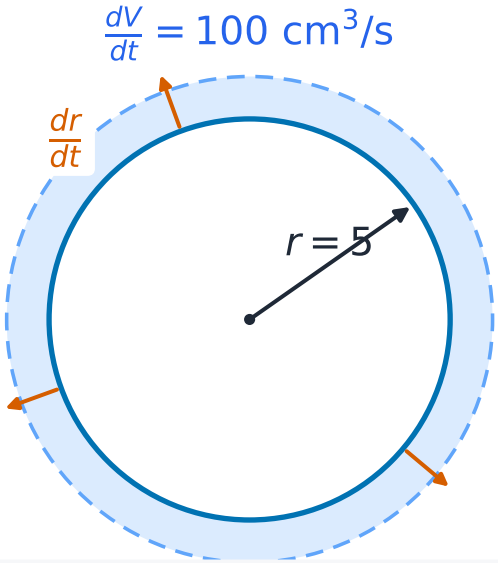
Worked example – the inflating balloon

$\frac{dV}{dt} = 100\ \mathrm{cm}^3/\mathrm{s}$: how fast is r growing at $r = 5$?

Start from $V = \frac{4}{3}\pi r^3$ and differentiate in t first:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100 = 4\pi(5)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{\pi} \approx 0.32\ \mathrm{cm/s}.$$



$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ (chain rule on $V = \frac{4}{3}\pi r^3$)

An inflating balloon links dV/dt and dr/dt through the chain rule

The classic related-rates error

✗ Substitute $r = 5$ first

$V = \frac{4}{3}\pi(5)^3$ is a constant, so $\frac{dV}{dt} = 0$. The radius is now frozen – the problem is destroyed.

✓ Differentiate first

Keep r a function of t, differentiate the general relationship, *then* put $r = 5$ in.

> Substituting the instant's value too early is the single most common lost mark here.

Exercises 2.1·2.3

2.1 State the first step in a related-rates problem. [1]

2.2 Differentiate $A = \pi r^2$ with respect to t. [2]

2.3 State what links the two rates. [1]

Exercises 2.1·2.3

2.1 State the first step in a related-rates problem. [1]

2.1

- write the relation connecting the quantities

2.2 Differentiate $A = \pi r^2$ with respect to t. [2]

2.2

- $\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$

2.3 State what links the two rates. [1]

2.3

- the derivative of the relation with respect to t

Exercises 3.1·3.2

3.1 In related rates, you differentiate the relation with respect to [1]

- **A** x
- **B** time t
- **C** r
- **D** nothing

3.2 Differentiating $A = \pi r^2$ with respect to t gives [1]

- **A** $2\pi r$
- **B** $2\pi r \dfrac{dr}{dt}$
- **C** πr^2
- **D** $2r$

Exercises 3.1·3.2

3.1 In related rates, you differentiate the relation with respect to [1]

- ☐ **A** x
- **B** time t
- **C** r
- **D** nothing

3.1 B

- time t.

3.2 Differentiating $A = \pi r^2$ with respect to t gives [1]

- ☐ **A** $2\pi r$
- **B** $2\pi r \dfrac{dr}{dt}$
- **C** πr^2
- **D** $2r$

3.2 B

- $2\pi r \dfrac{dr}{dt}$.

Exercise 3.3 (a)

3.3 $V = s^3$ (a cube).

(a) Differentiate with respect to t. [1]

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(a)

- $\dfrac{dV}{dt} = 3s^2 \dfrac{ds}{dt}$

Exercise 3.3 (b)

3.3 $V = s^3$ (a cube).

(b) State the chain-rule factor. [1]

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(b)

- $\dfrac{ds}{dt}$

Exercise 3.3 (c)

3.3 $V = s^3$ (a cube).

(c) Write $\dfrac{dV}{dt}$. [1]

Exercise 3.3 (c)

3.3 $V = s^3$ (a cube).

(c) Write $\dfrac{dV}{dt}$. [1]

(c)

- $\dfrac{dV}{dt} = 3s^2 \dfrac{ds}{dt}$