

Exercise walk-through

4.2

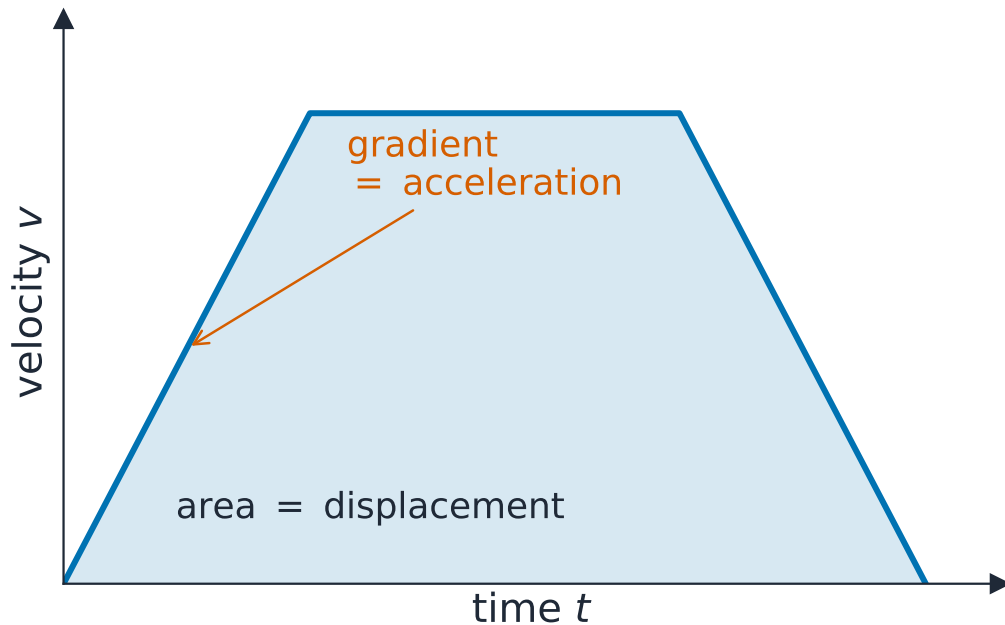
Straight-Line Motion: Connecting Position, Velocity, and Acceleration

AP Calculus BC

Straight-line motion

Three functions of time, linked by differentiation:

- **position** $s(t)$
- **velocity** $v(t) = s'(t)$ – signed
- **acceleration** $a(t) = v'(t) = s''(t)$



Reading the motion – the frequent exam parts

At rest

$$v(t) = 0$$

Direction

right/up if $v > 0$, left/down if $v < 0$

Changes direction

where v **changes sign**

Speed

$|v(t)|$ – no direction

A rising balloon: related rates link how fast radius, volume and height change together

Speeding up or slowing down?

Compare the **signs** of v and a :

Same sign $\Rightarrow$ speeding up. **Opposite** signs $\Rightarrow$ slowing down.

Velocity has direction; speed does not. The exam tests exactly this difference.



Related rates coaster

Worked example – a particle on a line

$$s(t) = t^3 - 6t^2 + 9t$$

$$v(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$$

At rest at $t = 1$ and $t = 3$; it changes direction at each. At $t = 2$: $v(2) = -3 < 0$ (moving left), and just after, $a > 0$ while $v < 0$ – opposite signs, so it is **slowing down**.

$$s(t) = t^3 - 6t^2 + 9t$$



$$v(t) = 3(t-1)(t-3)$$



$$a(t) = 6t - 12$$

Exercises 2.1·2.3

2.1 State velocity in terms of position.

[1]

2.2 State acceleration in terms of velocity.

[1]

2.3 For $x(t) = t^2 - 4t$, find $v(t)$ and the time at rest.

[2]

Exercises 2.1·2.3

2.1 State velocity in terms of position.

[1]

2.1

- $v(t) = x'(t)$
-

2.2 State acceleration in terms of velocity.

[1]

2.2

- $a(t) = v'(t)$
-

2.3 For $x(t) = t^2 - 4t$, find $v(t)$ and the time at rest.

[2]

2.3

- $v(t) = 2t - 4$; at rest when $2t - 4 = 0$, i.e. $t = 2$

Exercises 3.1-3.2

3.1 Velocity is the derivative of

[1]

☐

A acceleration

B position

C speed

D distance

3.2 The object is at rest when

[1]

☐

A $a = 0$

B $v = 0$

C $x = 0$

D $t = 0$

Exercises 3.1-3.2

3.1 Velocity is the derivative of

[1]

☐

A acceleration

B position

C speed

D distance

3.1 B

- position.

3.2 The object is at rest when

[1]

☐

A $a = 0$

B $v = 0$

C $x = 0$

D $t = 0$

3.2 B

- $v = 0$.

Exercise 3.3 (a)

3.3 $x(t) = t^2 - 4t$.

(a) Find $v(t)$.

[1]

Exercise 3.3 (a)

3.3 $x(t) = t^2 - 4t$.

(a) Find $v(t)$.

[1]

(a)

- $v(t) = 2t - 4$

Exercise 3.3 (b)

3.3 $x(t) = t^2 - 4t$.

(b) Find $a(t)$.

[1]

Exercise 3.3 (b)

3.3 $x(t) = t^2 - 4t$.

(b) Find $a(t)$.

[1]

(b)

- $a(t) = 2$

Exercise 3.3 (c)

3.3 $x(t) = t^2 - 4t$.

(c) Find when $v = 0$.

[1]

Exercise 3.3 (c)

3.3 $x(t) = t^2 - 4t$.

(c) Find when $v = 0$.

[1]

(c)

- $t = 2$