

Exercise walk-through

4.1

Interpreting the Meaning of the Derivative in Context

AP Calculus BC

Interpreting a derivative – units and a sentence

$f'(x)$ is the instantaneous rate of change of f per unit of x .

The **unit** of f' is $\text{unit of } f \div \text{unit of } x$: acres per week, litres per minute.

“Interpret $g'(140) = 2.3$ with units”

A full-credit sentence needs **four** parts:
At $x = 140$ (the moment), the quantity is increasing (the direction) *at about 2.3 units* (the value) **per unit** of x (the rate word).

Reading a derivative in context

what it is

$f'(x)$ is the instantaneous rate of change of f with respect to x

the units

the units of f divided by the units of x – and stating them is marked

the sign

positive means f is increasing; negative, decreasing

the second derivative

f'' gives concavity, and whether the rate itself is rising or falling

in motion

position, velocity, acceleration – and speed is $|v|$, which behaves differently from velocity

the sentence

“at $t = 3$ seconds the volume is increasing at 5 litres per second” – value, units, direction, context

An answer that gives a number without units and context scores less than half. Write the full sentence every time.

Reading a derivative in context, in detail

what it is

$f'(x)$ is the instantaneous rate of change of f with respect to x

the units

units of f divided by units of x – stating them is marked

beyond motion

the same idea models a draining tank, a spreading population, a cooling cup

the sign

positive means increasing, negative decreasing

an underestimate

a tangent-line approximation underestimates where the curve is concave up, and overestimates where it is concave down

the sentence

“at $t = 3$ minutes the volume is falling at 5 litres per minute” – value, units, direction, context

Concavity decides whether a linear approximation is high or low. That is the justification these questions want, not just the number.

Exercises 2.1-2.3

2.1 State the units of $f'(t)$ if f is in metres and t in seconds. [1]

2.2 State what a positive derivative means. [1]

2.3 If $P(t)$ is a population and $P'(5) = 200$, interpret this. [2]

Exercises 2.1-2.3

2.1 State the units of $f'(t)$ if f is in metres and t in seconds. [1]

2.1

- metres per second (m/s)
-

2.2 State what a positive derivative means. [1]

2.2

- the quantity is increasing
-

2.3 If $P(t)$ is a population and $P'(5) = 200$, interpret this. [2]

2.3

- at $t = 5$, the population is growing at 200 per unit time

Exercises 3.1-3.2

3.1 If $h(t)$ is a height in m and t in s, then $h'(t)$ has units

[1]

☐

A m

B m/s

C s/m

D m · s

3.2 A positive $f'(t)$ means the quantity is

[1]

☐

A decreasing

B increasing

C constant

D zero

Exercises 3.1-3.2

3.1 If $h(t)$ is a height in m and t in s, then $h'(t)$ has units

[1]

☐

A m

B m/s

C s/m

D m · s

3.1 B

- m/s.

3.2 A positive $f'(t)$ means the quantity is

[1]

☐

A decreasing

B increasing

C constant

D zero

3.2 B

- increasing.

Exercise 3.3 (a)

3.3 $W(t)$ is water in litres at time t (min), and $W'(3) = -4$.

(a) State the units.

[1]

Exercise 3.3 (a)

3.3 $W(t)$ is water in litres at time t (min), and $W'(3) = -4$.

(a) State the units.

[1]

(a)

- litres per minute (L/min)

Exercise 3.3 (b)

3.3 $W(t)$ is water in litres at time t (min), and $W'(3) = -4$.

(b) State what the sign means.

[1]

Exercise 3.3 (b)

3.3 $W(t)$ is water in litres at time t (min), and $W'(3) = -4$.

(b) State what the sign means.

[1]

(b)

- the amount is decreasing

Exercise 3.3 (c)

3.3 $W(t)$ is water in litres at time t (min), and $W'(3) = -4$.

(c) Interpret $W'(3) = -4$.

[1]

Exercise 3.3 (c)

3.3 $W(t)$ is water in litres at time t (min), and $W'(3) = -4$.

(c) Interpret $W'(3) = -4$.

[1]

(c)

- at $t = 3$ min, the water is falling at 4 L/min