

Exercise walk-through

3.2

Implicit Differentiation

AP Calculus BC

Implicit differentiation

Some curves are defined **implicitly**

– an equation in x and y not solved for y ,
e.g. $x^2 + y^2 = 25$.

- 1 differentiate **both sides** in x
- 2 each y -term picks up a $\frac{dy}{dx}$ (chain rule)
- 3 solve algebraically for $\frac{dy}{dx}$

$$x^2 + y^2 = 25$$



$$2x + 2y \frac{dy}{dx} = 0$$



$$\frac{dy}{dx} = -\frac{x}{y}$$

Worked example – tangent to a circle

Tangent to $x^2 + y^2 = 25$ at $(3, 4)$

From $\frac{dy}{dx} = -\frac{x}{y}$, at $(3, 4)$:

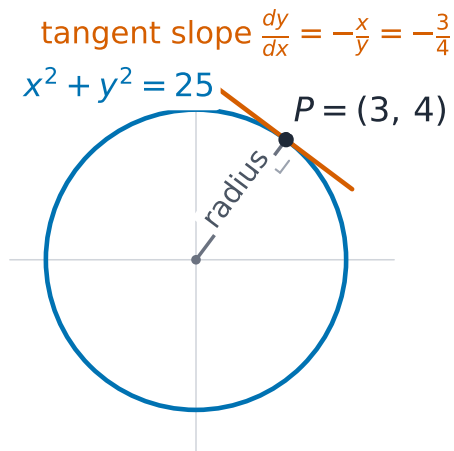
$$\frac{dy}{dx} = -\frac{3}{4}.$$

So the tangent is

$$y - 4 = -\frac{3}{4}(x - 3).$$

The tangent is **perpendicular to the radius** – a free sanity check.

Worked example – tangent to a circle, in detail



$$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} \cdot \text{tangent} \perp \text{radius}$$

Implicit differentiation of $x^2 + y^2 = 25$ gives $\frac{dy}{dx} = -\frac{x}{y}$, so at $(3, 4)$ the gradient is $-\frac{3}{4}$ – perpendicular to the radius.

Exam skill – “Show that $\frac{dy}{dx} = \dots$ ”

The target is **given**, so the marks are in the **algebra**, not the answer:

- 1 differentiate both sides
- 2 product/chain rule on mixed xy terms
- 3 collect every $\frac{dy}{dx}$ on one side
- 4 factor, then divide

A correct final line that **skips** the algebra earns little.

Horizontal tangent

$$\frac{dy}{dx} = 0: \text{numerator} = 0$$

Vertical tangent

$$\text{denominator} = 0$$

Exercises 2.1-2.3

2.1 State what to attach when differentiating y .

[1]

2.2 Differentiate $x^2 + y^2 = 25$ implicitly.

[2]

2.3 State the final step.

[1]

Exercises 2.1-2.3

2.1 State what to attach when differentiating y .

[1]

2.1

- the factor $\frac{dy}{dx}$ (i.e. y')

2.2 Differentiate $x^2 + y^2 = 25$ implicitly.

[2]

2.2

- $2x + 2y y' = 0$, so $y' = -\frac{x}{y}$

2.3 State the final step.

[1]

2.3

- solve the equation for $\frac{dy}{dx}$

Exercises 3.1-3.2

3.1 In implicit differentiation, differentiating y^2 gives

[1]

☐

A $2y$

B $2y y'$

C y'

D $2x$

3.2 After differentiating, you

[1]

☐

A stop

B solve for $\frac{dy}{dx}$

C set $y = 0$

D integrate

Exercises 3.1-3.2

3.1 In implicit differentiation, differentiating y^2 gives

[1]

☐

A $2y$

B $2y y'$

C y'

D $2x$

3.1 B

- $2y y'$.

3.2 After differentiating, you

[1]

☐

A stop

B solve for $\frac{dy}{dx}$

C set $y = 0$

D integrate

3.2 B

- solve for $\frac{dy}{dx}$.

Exercise 3.3 (a)

3.3 $x^2 + y^2 = 25$.

(a) Differentiate both sides.

[1]

Exercise 3.3 (a)

3.3 $x^2 + y^2 = 25$.

(a) Differentiate both sides.

[1]

(a)

- $2x + 2y y' = 0$

Exercise 3.3 (b)

3.3 $x^2 + y^2 = 25$.

(b) Isolate the y' term.

[1]

Exercise 3.3 (b)

3.3 $x^2 + y^2 = 25$.

(b) Isolate the y' term.

[1]

(b)

- $2y y' = -2x$

Exercise 3.3 (c)

3.3 $x^2 + y^2 = 25$.

(c) Solve for y' .

[1]

Exercise 3.3 (c)

3.3 $x^2 + y^2 = 25$.

(c) Solve for y' .

[1]

(c)

- $y' = -\frac{x}{y}$