

Exercise walk-through

2.7

Derivatives of $\cos x$, $\sin x$, e^x , and $\ln x$

AP Calculus BC

A limit that is really a derivative

If a limit has the **shape** of the definition

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h},$$

do not expand it – just evaluate $f'(a)$.

Evaluate the limit

$$\lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} + h\right) - 1}{h}$$

This is $\frac{d}{dx} \sin x \Big|_{x=\pi/2} = \cos \frac{\pi}{2} = 0$.

The product rule

A product is **not** the product of the derivatives. Use the **product rule** :

$$(uv)' = u'v + uv'.$$

“Derivative of the first times the second, **plus** the first times the derivative of the second.”

Differentiate $f(x) = x^2 \sin x$

$u = x^2$ ($u' = 2x$), $v = \sin x$ ($v' = \cos x$):

$$f'(x) = 2x \sin x + x^2 \cos x.$$

Read the **structure** first – a product needs the product rule.

Unit 2 – key takeaways

1

The derivative

the limit of the secant slope – the tangent's gradient

2

Existence

differentiable $\Rightarrow$ continuous;
corners and vertical tangents fail

3

The rules

power, constant multiple, sum;
product and quotient

4

Building blocks

$\sin$, $\cos$, e^x , $\ln x$ – know them by heart

Exercises 2.1·2.3

2.1 State $\frac{d}{dx}[\sin x]$.

[1]

2.2 State $\frac{d}{dx}[e^x]$.

[1]

2.3 State $\frac{d}{dx}[\cos x]$ and $\frac{d}{dx}[\ln x]$.

[2]

Exercises 2.1-2.3

2.1 State $\frac{d}{dx}[\sin x]$.

[1]

2.1

- $\cos x$
-

2.2 State $\frac{d}{dx}[e^x]$.

[1]

2.2

- e^x
-

2.3 State $\frac{d}{dx}[\cos x]$ and $\frac{d}{dx}[\ln x]$.

[2]

2.3

- $\frac{d}{dx}[\cos x] = -\sin x$; $\frac{d}{dx}[\ln x] = \frac{1}{x}$

Exercise 3.1

3.1 $\frac{d}{dx}[\cos x]$ equals

[1]

☐

A $\sin x$

B $-\sin x$

C $\cos x$

D $-\cos x$

Exercise 3.1

3.1 $\frac{d}{dx}[\cos x]$ equals

[1]

☐

A $\sin x$

B $-\sin x$

C $\cos x$

D $-\cos x$

3.1 B

- $-\sin x$.

Exercise 3.2

3.2 $\frac{d}{dx}[\ln x]$ equals

[1]

☐

A x

B $\frac{1}{x}$

C $\ln x$

D e^x

Exercise 3.2

3.2 $\frac{d}{dx}[\ln x]$ equals

[1]

☐

A x

B $\frac{1}{x}$

C $\ln x$

D e^x

3.2 B

• $\frac{1}{x}$.

Exercise 3.3 (a)

3.3 Differentiate each.

(a) $\sin x$.

[1]

Exercise 3.3 (a)

3.3 Differentiate each.

(a) $\sin x$.

[1]

(a)

- $\cos x$

Exercise 3.3 (b)

3.3 Differentiate each.

(b) e^x .

[1]

Exercise 3.3 (b)

3.3 Differentiate each.

(b) e^x .

[1]

(b)

- e^x

Exercise 3.3 (c)

3.3 Differentiate each.

(c) $\cos x$.

[1]

Exercise 3.3 (c)

3.3 Differentiate each.

(c) $\cos x$.

[1]

(c)

- $-\sin x$