

Exercise walk-through

2.6

Derivative Rules: Constant, Sum, Difference, and Constant Multiple

AP Calculus BC

The power rule

The **power rule** handles **any** real power:

$$\frac{d}{dx} x^r = r x^{r-1}.$$

Rewrite as a power **first**: $\frac{1}{x} = x^{-1}$,
 $\sqrt{x} = x^{1/2}$.

$$x^5 \rightarrow 5x^4$$

$$\frac{1}{x} = x^{-1} \rightarrow -x^{-2}$$

$$\sqrt{x} = x^{1/2} \rightarrow \frac{1}{2}x^{-1/2}$$

Constant, multiple, sum & difference

Constant

$$\frac{d}{dx} k = 0$$

Constant multiple

$$\frac{d}{dx} (kf) = kf'$$

Sum / difference

$$(f \pm g)' = f' \pm g'$$

Differentiate a polynomial term by term

$$\frac{d}{dx} (4x^3 - 5x + 7) = 12x^2 - 5.$$

The constant 7 contributes nothing – it does not change.

Four building blocks to know by heart

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

Note the **minus sign** on $\cos$, and that e^x is its **own** derivative.

A limit that is really a derivative

If a limit has the **shape** of the definition

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h},$$

do not expand it – just evaluate $f'(a)$.

Evaluate the limit

$$\lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} + h\right) - 1}{h}$$

This is $\frac{d}{dx} \sin x \Big|_{x=\pi/2} = \cos \frac{\pi}{2} = 0$.

The product rule

A product is **not** the product of the derivatives. Use the **product rule** :

$$(uv)' = u'v + uv'.$$

“Derivative of the first times the second, **plus** the first times the derivative of the second.”

Differentiate $f(x) = x^2 \sin x$

$u = x^2$ ($u' = 2x$), $v = \sin x$ ($v' = \cos x$):

$$f'(x) = 2x \sin x + x^2 \cos x.$$

Read the **structure** first – a product needs the product rule.

Exercises 2.1-2.3

2.1 State the sum rule.

[1]

2.2 Differentiate $3x^2$.

[1]

2.3 Differentiate $x^3 + x$.

[2]

Exercises 2.1-2.3

2.1 State the sum rule.

[1]

2.1

- $\frac{d}{dx}[f + g] = f' + g'$

2.2 Differentiate $3x^2$.

[1]

2.2

- $6x$

2.3 Differentiate $x^3 + x$.

[2]

2.3

- $3x^2 + 1$

Exercises 3.1·3.2

3.1 $\frac{d}{dx}[5x^2]$ equals

[1]

☐

A $5x$

B $10x$

C $5x^2$

D $10x^2$

3.2 $\frac{d}{dx}[f + g]$ equals

[1]

☐

A $f' \cdot g'$

B $f' + g'$

C $f' - g'$

D fg

Exercises 3.1-3.2

3.1 $\frac{d}{dx}[5x^2]$ equals

[1]

☐

A $5x$

B $10x$

C $5x^2$

D $10x^2$

3.1 B

- $10x$.

3.2 $\frac{d}{dx}[f + g]$ equals

[1]

☐

A $f' \cdot g'$

B $f' + g'$

C $f' - g'$

D fg

3.2 B

- $f' + g'$.

Exercise 3.3 (a)

3.3 $y = 4x^3 - 2x$.

(a) Differentiate $4x^3$.

[1]

Exercise 3.3 (a)

3.3 $y = 4x^3 - 2x$.

(a) Differentiate $4x^3$.

[1]

(a)

- $12x^2$

Exercise 3.3 (b)

3.3 $y = 4x^3 - 2x$.

(b) Differentiate $-2x$.

[1]

Exercise 3.3 (b)

3.3 $y = 4x^3 - 2x$.

(b) Differentiate $-2x$.

[1]

(b)

- -2

Exercise 3.3 (c)

3.3 $y = 4x^3 - 2x$.

(c) State y' .

[1]

Exercise 3.3 (c)

3.3 $y = 4x^3 - 2x$.

(c) State y' .

[1]

(c)

- $y' = 12x^2 - 2$