

Exercise walk-through

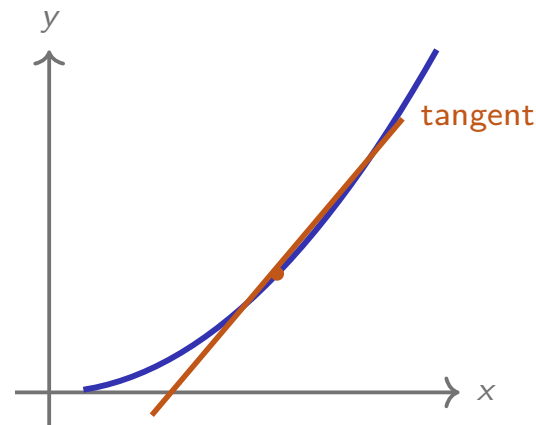
2.1

Defining Average and Instantaneous Rates of Change at a Point

AP Calculus BC

What we will cover

- 1 Average vs instantaneous rate
- 2 The derivative as a limit
- 3 When a derivative exists
- 4 Basic derivative rules
- 5 Product & quotient rules
- 6 Trig, exp & log derivatives



The notations, and the tangent line

the notations $\frac{dy}{dx}$, $f'(x)$, y' – all naming the same object

which to use Leibniz $\frac{dy}{dx}$ when the variables matter; Lagrange $f'(x)$ when the function does

the second derivative $\frac{d^2y}{dx^2}$ or $f''(x)$

the tangent line give it as $y - f(a) = f'(a)(x - a)$

why that form it needs no rearrangement and no risk of an arithmetic slip in the intercept

the check substitute $x = a$: the equation must give $y = f(a)$

Writing the tangent in point-slope form is a marked convention, not a preference. Converting it to $y = mx + c$ adds a step and an opportunity to lose a mark.

Notation, and what differentiability guarantees

the notations

$\frac{dy}{dx}$, $f'(x)$, y' – all the same object

what it is

the derivative at a point is the **slope of the tangent** there, and the instantaneous rate of change

the relationship

differentiable $\Rightarrow$ continuous

the converse is false

a continuous function can fail to be differentiable – at a corner, a cusp, or a vertical tangent

the standard example

$|x|$ at $x = 0$: continuous, and with no single tangent slope

the tangent line

give it as $y - f(a) = f'(a)(x - a)$

Differentiability is the **stronger** condition. On a justification question, say which direction of the implication you are using.

Exercises 2.1-2.3

2.1 State the difference quotient for the average rate over $[a, a + h]$. [1]

2.2 State what happens to it as $h \rightarrow 0$. [1]

2.3 For $f(x) = x^2$, compute $\frac{f(2+h) - f(2)}{h}$ and simplify. [2]

Exercises 2.1-2.3

2.1 State the difference quotient for the average rate over $[a, a + h]$. [1]

2.1

- $$\frac{f(a + h) - f(a)}{h}$$

2.2 State what happens to it as $h \rightarrow 0$. [1]

2.2

- it approaches the instantaneous rate of change (the derivative)

2.3 For $f(x) = x^2$, compute $\frac{f(2 + h) - f(2)}{h}$ and simplify. [2]

2.3

- $$\frac{(2 + h)^2 - 4}{h} = \frac{4h + h^2}{h} = 4 + h$$

Exercises 3.1-3.2

3.1 The average rate of change over $[a, a + h]$ is

[1]

☐

A $f(a)h$

B $\frac{f(a+h) - f(a)}{h}$

C $f'(a)$

D h

3.2 The instantaneous rate is the average rate as

[1]

☐

A $h \rightarrow \infty$

B $h \rightarrow 0$

C $h = 1$

D $a \rightarrow 0$

Exercises 3.1·3.2

3.1 The average rate of change over $[a, a + h]$ is

[1]

- ☐ **A** $f(a)h$
B $\frac{f(a+h) - f(a)}{h}$
C $f'(a)$
D h

3.1 B

- $\frac{f(a+h) - f(a)}{h}.$

3.2 The instantaneous rate is the average rate as

[1]

- ☐ **A** $h \rightarrow \infty$
B $h \rightarrow 0$
C $h = 1$
D $a \rightarrow 0$

3.2 B

- $h \rightarrow 0.$

Exercise 3.3 (a)

3.3 $f(x) = x^2$ at $a = 3$.

(a) Write $f(3 + h)$.

[1]

Exercise 3.3 (a)

3.3 $f(x) = x^2$ at $a = 3$.

(a) Write $f(3 + h)$.

[1]

(a)

- $(3 + h)^2$

Exercise 3.3 (b)

3.3 $f(x) = x^2$ at $a = 3$.

(b) Form the difference quotient.

[1]

Exercise 3.3 (b)

3.3 $f(x) = x^2$ at $a = 3$.

(b) Form the difference quotient.

[1]

(b)

- $\frac{(3+h)^2 - 9}{h} = 6 + h$

Exercise 3.3 (c)

3.3 $f(x) = x^2$ at $a = 3$.

(c) State its limit as $h \rightarrow 0$.

[1]

Exercise 3.3 (c)

3.3 $f(x) = x^2$ at $a = 3$.

(c) State its limit as $h \rightarrow 0$.

[1]

(c)

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