

Exercise walk-through

10.9

Determining Absolute or Conditional Convergence

AP Calculus BC

Convergent and divergent series

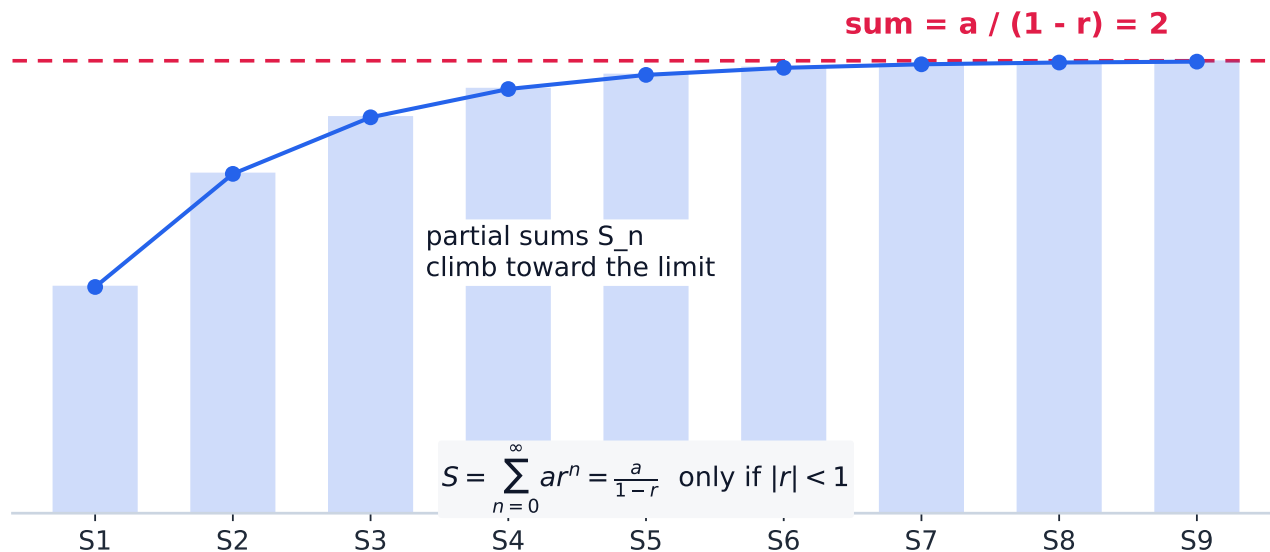
An **infinite series**

$$S_N = a_1 + \cdots + a_N.$$

$\sum a_n$ is defined as the limit of its **partial sums**

Every convergence question is really a question about **the limit of the partial sums**.

Convergent and divergent series, in detail



An **infinite series** $\sum a_n$ is defined as the limit of its **partial sums** $S_N = a_1 + \cdots + a_N$.

Geometric series – the one you can sum exactly

A **geometric series** has a constant ratio r . It converges **exactly** when $|r| < 1$:

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

It underlies **power series** later in the unit.

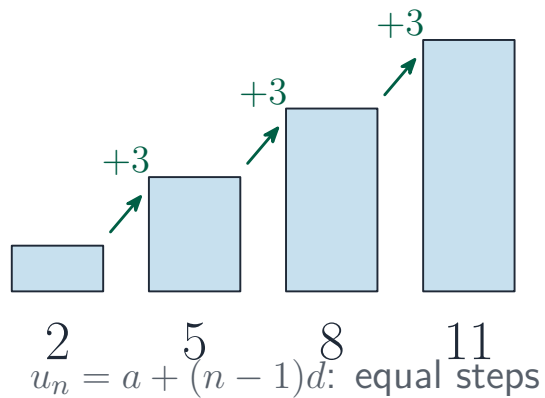
Sum $3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \dots$

Here $a = 3$ and $r = \frac{1}{2}$, and $|r| < 1$:

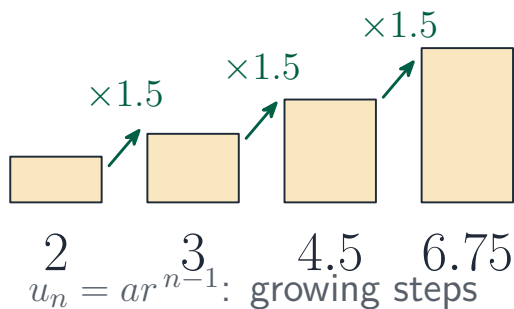
$$\frac{a}{1-r} = \frac{3}{1-\frac{1}{2}} = 6.$$

Working with Geometric Series

arithmetic: add d each step



geometric: multiply by r each step



A geometric sequence multiplies by the same ratio at each step

Exercises 2.1-2.3

2.1 For $\sum \frac{(-1)^n}{n^2}$, does $\sum \left| \frac{(-1)^n}{n^2} \right|$ converge? [1]

2.2 Classify $\sum \frac{(-1)^n}{n^2}$ (absolute / conditional). [1]

2.3 For $\sum \frac{(-1)^n}{n}$, does $\sum \frac{1}{n}$ converge? What does that make the series? [2]

Exercises 2.1-2.3

2.1 For $\sum \frac{(-1)^n}{n^2}$, does $\sum \left| \frac{(-1)^n}{n^2} \right|$ converge? [1]

2.1

- Yes – $\sum \frac{1}{n^2}$ is a convergent p-series

2.2 Classify $\sum \frac{(-1)^n}{n^2}$ (absolute / conditional). [1]

2.2

- Absolutely convergent

2.3 For $\sum \frac{(-1)^n}{n}$, does $\sum \frac{1}{n}$ converge? What does that make the series? [2]

2.3

- $\sum \frac{1}{n}$ diverges; so $\sum \frac{(-1)^n}{n}$ is **conditionally** convergent

Exercise 3.1

3.1 A series is **conditionally** convergent if

[1]

☐

A both $\sum a_n$ and $\sum |a_n|$ converge

B $\sum a_n$ converges but $\sum |a_n|$ diverges

C both diverge

D $\sum |a_n|$ converges but $\sum a_n$ diverges

Exercise 3.1

3.1 A series is **conditionally** convergent if

[1]



A both $\sum a_n$ and $\sum |a_n|$ converge

B $\sum a_n$ converges but $\sum |a_n|$ diverges

C both diverge

D $\sum |a_n|$ converges but $\sum a_n$ diverges

3.1 B

- converges, but not absolutely

Exercise 3.2 (a)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(a) Test $\sum \frac{1}{\sqrt{n}}$ for convergence.

[2]

Exercise 3.2 (a)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(a) Test $\sum \frac{1}{\sqrt{n}}$ for convergence.

[2]

(a)

- $\sum \frac{1}{\sqrt{n}}$ is a p-series with $p = \frac{1}{2} \leq 1$, diverges

Exercise 3.2 (b)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(b) Test the alternating series itself.

[1]

Exercise 3.2 (b)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(b) Test the alternating series itself.

[1]

(b)

- Alternating series test: $b_n = \frac{1}{\sqrt{n}}$ decreasing $\rightarrow 0$, so it converges

Exercise 3.2 (c)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(c) Classify the series.

[1]

Exercise 3.2 (c)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(c) Classify the series.

[1]

(c)

- Conditionally convergent

Exercise 3.3

3.3 Explain why absolute convergence is “stronger” than conditional convergence. [2]

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3.3

- Absolute convergence guarantees the series converges no matter how the terms are arranged, and it implies ordinary convergence; conditional convergence depends on the sign pattern and can be broken by rearrangement