

Exercise walk-through

10.3

The n th Term Test for Divergence

AP Calculus BC

Start here – the n th term test

If the terms do not shrink to zero, the sum cannot settle:

$$\lim_{n \rightarrow \infty} a_n \neq 0 \implies \text{diverges.}$$

Always check this **quick test first**.

✗ It is **only** a test for **divergence**

If $a_n \rightarrow 0$ the test is **inconclusive** – the series may **still** diverge

p -series and the harmonic series

$$\sum \frac{1}{n^p} \begin{cases} \text{converges} & p > 1 \\ \text{diverges} & p \leq 1 \end{cases}$$

The **integral test** proves this:
for positive, decreasing, continuous f ,
 $\sum a_n$ and $\int_1^\infty f$ **both** converge or **both** diverge.

The **harmonic series** $\sum \frac{1}{n}$
($p = 1$) **diverges** – even though its
terms go to zero. A famous, must-
know fact.

The p -series family is the standard **yardstick**
for the comparison tests.

Exam tips

- Test a series for convergence with the right tool: **geometric** ($|r| < 1$, $\sum \frac{a}{1-r}$), n th-term, ratio, integral, comparison, or alternating-series test.
- A **geometric** infinite sum converges only when $|r| < 1$; otherwise it diverges.
- Build a **Taylor/Maclaurin** series to approximate a function; more terms give a better fit **near the centre**.
- Know the standard Maclaurin series for e^x , $\sin x$, $\cos x$, and $\frac{1}{1-x}$.
- Find the **radius/interval of convergence** with the ratio test, then check the endpoints separately.

Exercises 2.1-2.3

2.1 Find $\lim_{n \rightarrow \infty} \frac{2n}{3n+1}$.

[2]

2.2 State whether $\sum \frac{2n}{3n+1}$ converges or diverges, with a reason.

[2]

2.3 For $\sum \frac{1}{n}$, the terms go to 0. Does the n th term test prove convergence?

[1]

Exercises 2.1·2.3

2.1 Find $\lim_{n \rightarrow \infty} \frac{2n}{3n+1}$.

[2]

2.1

- $\frac{2n}{3n+1} \rightarrow \frac{2}{3}$

2.2 State whether $\sum \frac{2n}{3n+1}$ converges or diverges, with a reason.

[2]

2.2

- Diverges – the limit is $\frac{2}{3} \neq 0$, so by the n th term test it diverges

2.3 For $\sum \frac{1}{n}$, the terms go to 0. Does the n th term test prove convergence?

[1]

2.3

- No – the test is inconclusive when the terms go to 0

Exercise 3.1

3.1 If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum a_n$

[1]

☐

A converges

B diverges

C may converge or diverge (test inconclusive)

D equals 0

Exercise 3.1

3.1 If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum a_n$

[1]

☐

A converges

B diverges

C may converge or diverge (test inconclusive)

D equals 0

3.1 C

- the test is inconclusive when $a_n \rightarrow 0$

Exercise 3.2 (a)

3.2 Consider $\sum_{n=1}^{\infty} \frac{n^2}{2n^2 + 5}$.

(a) Find $\lim_{n \rightarrow \infty} a_n$.

[2]

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(a) Find $\lim_{n \rightarrow \infty} a_n$.

[2]

(a)

- $\frac{n^2}{2n^2 + 5} \rightarrow \frac{1}{2}$

Exercise 3.2 (b)

3.2 Consider $\sum_{n=1}^{\infty} \frac{n^2}{2n^2 + 5}$.

(b) State the conclusion of the n th term test.

[2]

Exercise 3.2 (b)

3.2 Consider $\sum_{n=1}^{\infty} \frac{n^2}{2n^2 + 5}$.

(b) State the conclusion of the n th term test.

[2]

(b)

- Limit $\neq 0$, so the series **diverges**

Exercise 3.3

3.3 A student claims that because the terms of $\sum \frac{1}{\sqrt{n}}$ tend to 0, the series converges. Explain the error. [2]

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3.3

- The n th term test only detects **divergence**; $a_n \rightarrow 0$ does not prove convergence (in fact $\sum \frac{1}{\sqrt{n}}$ is a p -series with $p = \frac{1}{2} \leq 1$ and diverges)