

Exercise walk-through

10.2

Working with Geometric Series

AP Calculus BC

You must be able to

- identify the first term a and common **ratio** r
- apply the convergence test $|r| < 1$
- compute the sum $\frac{a}{1 - r}$

1

The method

Method — The geometric sum

A geometric series $\sum_{n=0}^{\infty} ar^n$ converges **exactly when** $|r| < 1$, and then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

Exercises 2.1·2.2

2.1 State a and r for $\sum_{n=0}^{\infty} 5 \left(\frac{1}{3}\right)^n$. [2]

2.2 Find the sum of $\sum_{n=0}^{\infty} 5 \left(\frac{1}{3}\right)^n$. [2]

Exercises 2.1·2.2

2.1 State a and r for $\sum_{n=0}^{\infty} 5 \left(\frac{1}{3}\right)^n$. [2]

2.1

- $a = 5, r = \frac{1}{3}$

2.2 Find the sum of $\sum_{n=0}^{\infty} 5 \left(\frac{1}{3}\right)^n$. [2]

2.2

- $\frac{5}{1 - 1/3} = \frac{5}{2/3} = \frac{15}{2}$

Exercises 2.3-3.1

2.3 Does $\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$ converge? Give a reason. [1]

3.1 A geometric series $\sum ar^n$ converges when [1]

☐

A $r > 0$

B $|r| < 1$

C $|r| > 1$

D $a < 1$

Exercises 2.3-3.1

2.3 Does $\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$ converge? Give a reason. [1]

2.3

- No — $|r| = \frac{5}{4} > 1$, so it diverges

3.1 A geometric series $\sum ar^n$ converges when [1]

☐

A $r > 0$

B $|r| < 1$

C $|r| > 1$

D $a < 1$

3.1 B

- converges iff $|r| < 1$

Exercise 3.2 (a)

3.2 Consider $\sum_{n=0}^{\infty} 6 \left(-\frac{1}{2}\right)^n$.

(a) State a and r .

[2]

Exercise 3.2 (a)

3.2 Consider $\sum_{n=0}^{\infty} 6 \left(-\frac{1}{2}\right)^n$.

(a) State a and r .

[2]

(a)

- $a = 6, r = -\frac{1}{2}$

Exercise 3.2 (b)

3.2 Consider $\sum_{n=0}^{\infty} 6 \left(-\frac{1}{2}\right)^n$.

(b) Find the sum.

[2]

Exercise 3.2 (b)

3.2 Consider $\sum_{n=0}^{\infty} 6 \left(-\frac{1}{2}\right)^n$.

(b) Find the sum.

[2]

(b)

- $$\frac{6}{1 - (-\frac{1}{2})} = \frac{6}{3/2} = 4$$

Exercise 3.3

3.3 Find the sum of $\sum_{n=1}^{\infty} 4 \left(\frac{2}{5}\right)^n$ (note the index starts at 1).

[3]

$$a_n = 4 \left(\frac{2}{5}\right)^n$$

$$n = 1, 2, 3, \dots$$

Exercise 3.3

3.3 Find the sum of $\sum_{n=1}^{\infty} 4 \left(\frac{2}{5}\right)^n$ (note the index starts at 1). [3]

$$a_n = 4 \left(\frac{2}{5}\right)^n$$

$$n = 1, 2, 3, \dots$$

3.3

- First term ($n = 1$) is $4 \cdot \frac{2}{5} = \frac{8}{5}$, ratio $\frac{2}{5}$; $\text{sum} = \frac{8/5}{1 - 2/5} = \frac{8/5}{3/5} = \frac{8}{3}$