

Exercise walk-through

10.13

Radius and Interval of Convergence of Power Series

AP Calculus BC

Convergent and divergent series

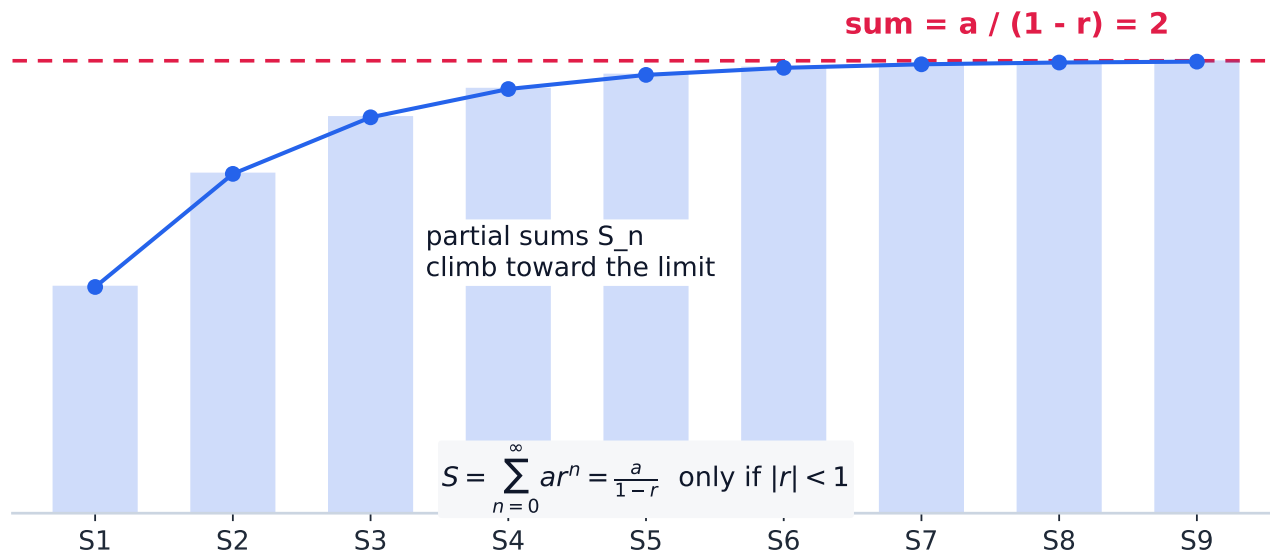
An **infinite series**

$$S_N = a_1 + \cdots + a_N.$$

$\sum a_n$ is defined as the limit of its **partial sums**

Every convergence question is really a question about **the limit of the partial sums**.

Convergent and divergent series, in detail



An **infinite series** $\sum a_n$ is defined as the limit of its **partial sums** $S_N = a_1 + \cdots + a_N$.

Geometric series – the one you can sum exactly

A **geometric series** has a constant ratio r . It converges **exactly** when $|r| < 1$:

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

It underlies **power series** later in the unit.

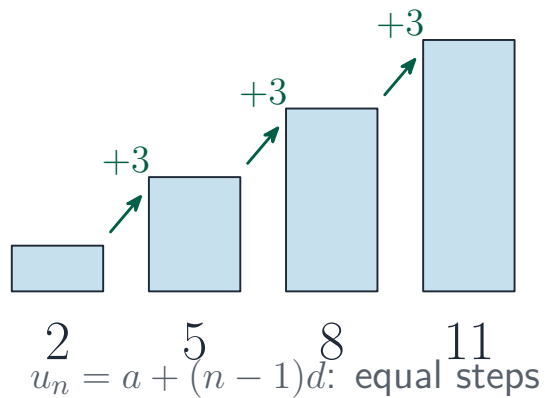
Sum $3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \dots$

Here $a = 3$ and $r = \frac{1}{2}$, and $|r| < 1$:

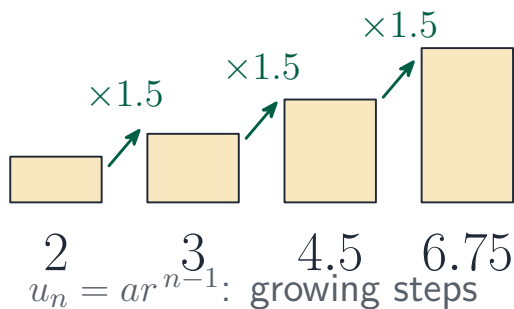
$$\frac{a}{1-r} = \frac{3}{1-\frac{1}{2}} = 6.$$

Working with Geometric Series

arithmetic: add d each step



geometric: multiply by r each step



A geometric sequence multiplies by the same ratio at each step

Exercises 2.1-2.3

2.1 For $\sum \frac{x^n}{2^n}$, apply the ratio test to find R . [3]

2.2 State the open interval of convergence for a series centered at 0 with $R = 2$. [1]

2.3 Why must the endpoints be tested separately? [1]

Exercises 2.1·2.3

2.1 For $\sum \frac{x^n}{2^n}$, apply the ratio test to find R . [3]

2.1

- $\left| \frac{x^{n+1}/2^{n+1}}{x^n/2^n} \right| = \frac{|x|}{2}$; set < 1 : $|x| < 2$; $R = 2$

2.2 State the open interval of convergence for a series centered at 0 with $R = 2$. [1]

2.2

- $(-2, 2)$

2.3 Why must the endpoints be tested separately? [1]

2.3

- The ratio test gives $L = 1$ at the endpoints, so it is inconclusive there

Exercise 3.1

3.1 The radius of convergence is found using the

[1]

☐

A integral test

B ratio test

C n th term test

D comparison test

Exercise 3.1

3.1 The radius of convergence is found using the

[1]

☐

A integral test

B ratio test

C n th term test

D comparison test

3.1 B

- the ratio test gives the radius

Exercise 3.2 (a)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$.

(a) Find the radius of convergence.

[3]

Exercise 3.2 (a)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$.

(a) Find the radius of convergence.

[3]

(a)

$$\bullet \left| \frac{(x-1)^{n+1}/(n+1)}{(x-1)^n/n} \right| = |x-1| \cdot \frac{n}{n+1} \rightarrow |x-1|; < 1 \Rightarrow R = 1$$

Exercise 3.2 (b)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$.

(b) Test the endpoints $x = 0$ and $x = 2$.

[2]

Exercise 3.2 (b)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$.

(b) Test the endpoints $x = 0$ and $x = 2$.

[2]

(b)

- $x = 2$: $\sum \frac{1}{n}$ diverges; $x = 0$: $\sum \frac{(-1)^n}{n}$ converges

Exercise 3.2 (c)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$.

(c) State the interval of convergence.

[1]

Exercise 3.2 (c)

3.2 Consider $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$.

(c) State the interval of convergence.

[1]

(c)

- $[0, 2)$

Exercise 3.3

3.3 Find the radius of convergence of $\sum \frac{x^n}{n!}$.

[3]

Exercise 3.3

3.3 Find the radius of convergence of $\sum \frac{x^n}{n!}$. [3]

3.3

- $\left| \frac{x^{n+1}/(n+1)!}{x^n/n!} \right| = \frac{|x|}{n+1} \rightarrow 0 < 1$ for all x ; $R = \infty$ (converges everywhere)