

Exercise walk-through

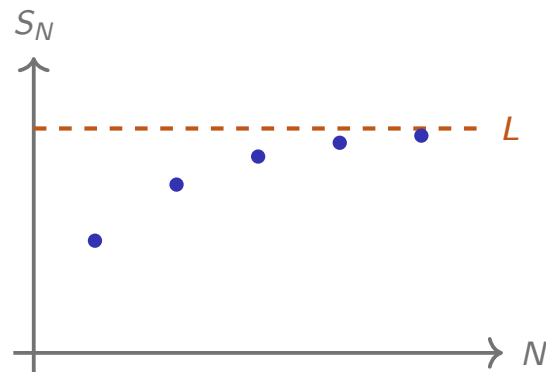
10.12

Lagrange Error Bound

AP Calculus BC

What we will cover

- 1 Convergence & partial sums
- 2 Geometric series
- 3 The convergence tests
- 4 Absolute vs conditional
- 5 Taylor polynomials & error
- 6 Power series & intervals



Absolute vs conditional convergence

Absolutely

$\sum |a_n|$ converges –
the **stronger** property

Conditionally

$\sum a_n$ converges but
 $\sum |a_n|$ does not

The classic: $\sum \frac{(-1)^n}{n}$ converges **conditionally** – it relies on the **cancellation of signs**, since $\sum \frac{1}{n}$ diverges.

Alternating error bound

$$|S - S_N| \leq b_{N+1}$$

The error is **no larger than the first omitted term**.

The Lagrange error bound

How far can a Taylor polynomial be from the truth?

$$|R_n(x)| \leq \frac{\max |f^{(n+1)}(z)|}{(n+1)!} |x - a|^{n+1}.$$

Bound the $(n+1)$ th derivative on the interval, then compute – the standard way to **prove** an estimate is accurate enough.

Exercises 2.1·2.3

2.1 State the Lagrange error bound.

[1]

2.2 For $f(x) = \cos x$, state a bound for $|f^{(n+1)}(z)|$.

[1]

2.3 Compute $\frac{1}{3!}(0.1)^3$.

[2]

Exercises 2.1-2.3

2.1 State the Lagrange error bound.

[1]

2.1

- $|R_n(x)| \leq \frac{\max |f^{(n+1)}(z)|}{(n+1)!} |x - a|^{n+1}$

2.2 For $f(x) = \cos x$, state a bound for $|f^{(n+1)}(z)|$.

[1]

2.2

- $|f^{(n+1)}(z)| \leq 1$ (derivatives of $\cos$ are bounded by 1)

2.3 Compute $\frac{1}{3!}(0.1)^3$.

[2]

2.3

- $\frac{1}{6}(0.001) = \frac{0.001}{6} \approx 1.67 \times 10^{-4}$

Exercise 3.1

3.1 In the Lagrange error bound, the factor $(n + 1)!$ appears in the

[1]

☐

A numerator

B denominator

C exponent

D derivative

Exercise 3.1

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[1]

☐

A numerator

B denominator

C exponent

D derivative

3.1 B

- $(n + 1)!$ is in the denominator

Exercise 3.2 (a)

3.2 The function $f(x) = e^x$ is approximated near 0 by its 2nd-degree Maclaurin polynomial, for $0 \leq x \leq 0.5$. On this interval $|f'''(z)| \leq e^{0.5} < 2$.

(a) Write the Lagrange bound for $|R_2(0.5)|$. [2]

Exercise 3.2 (a)

3.2 The function $f(x) = e^x$ is approximated near 0 by its 2nd-degree Maclaurin polynomial, for $0 \leq x \leq 0.5$. On this interval $|f'''(z)| \leq e^{0.5} < 2$.

(a) Write the Lagrange bound for $|R_2(0.5)|$. [2]

(a)

- $|R_2(0.5)| \leq \frac{\max |f'''|}{3!} (0.5)^3$

Exercise 3.2 (b)

3.2 The function $f(x) = e^x$ is approximated near 0 by its 2nd-degree Maclaurin polynomial, for $0 \leq x \leq 0.5$. On this interval $|f'''(z)| \leq e^{0.5} < 2$.

(b) Evaluate the bound using $|f'''| \leq 2$.

[2]

Exercise 3.2 (b)

3.2 The function $f(x) = e^x$ is approximated near 0 by its 2nd-degree Maclaurin polynomial, for $0 \leq x \leq 0.5$. On this interval $|f'''(z)| \leq e^{0.5} < 2$.

(b) Evaluate the bound using $|f'''| \leq 2$.

[2]

(b)

- $\leq \frac{2}{6}(0.125) = \frac{0.25}{6} \approx 0.0417$

Exercise 3.3

3.3 Bound the error when $\sin(0.3)$ is approximated by the 1st-degree Maclaurin polynomial $P_1(x) = x$. [3]

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3.3

- $n = 1$, $|f''(z)| = |-\sin z| \leq 1$; $|R_1(0.3)| \leq \frac{1}{2!}(0.3)^2 = \frac{0.09}{2} = 0.045$