

Exercise walk-through

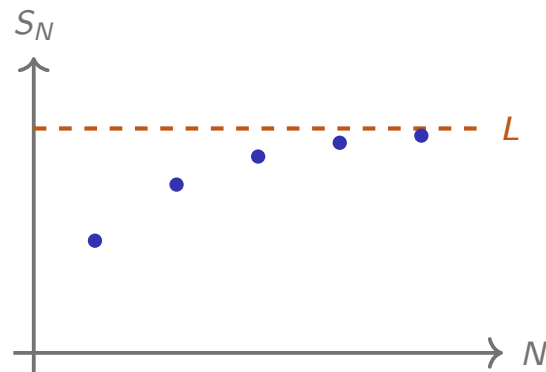
10.10

Alternating Series Error Bound

AP Calculus BC

What we will cover

- 1 Convergence & partial sums
- 2 Geometric series
- 3 The convergence tests
- 4 Absolute vs conditional
- 5 Taylor polynomials & error
- 6 Power series & intervals



Absolute vs conditional convergence

Absolutely

$\sum |a_n|$ converges –
the **stronger** property

Conditionally

$\sum a_n$ converges but
 $\sum |a_n|$ does not

The classic: $\sum \frac{(-1)^n}{n}$ converges **conditionally** – it relies on the **cancellation of signs**, since $\sum \frac{1}{n}$ diverges.

Alternating error bound

$$|S - S_N| \leq b_{N+1}$$

The error is **no larger** than the first omitted term.

Absolute against conditional convergence

Converges absolutely

A series **converges absolutely** when $\sum |a_n|$ converges – and then $\sum a_n$ converges too, whatever the signs do.

Converges conditionally

It **converges conditionally** when $\sum a_n$ converges but $\sum |a_n|$ does not – the alternating harmonic series is the standard case.

Why it matters

An absolutely convergent series can be rearranged freely; a conditionally convergent one can be rearranged to give *any* sum at all.

How to test

Test $\sum |a_n|$ first. If it converges you are finished; if not, try the alternating series test on $\sum a_n$.

“Converges” is not one property. Say which kind, because the error bound and the rearrangement both depend on it.

Exercises 2.1·2.3

2.1 State the alternating series error bound.

[1]

2.2 For $\sum \frac{(-1)^{n+1}}{n}$ approximated by S_5 , bound the error.

[2]

2.3 For $\sum \frac{(-1)^n}{n^2}$ approximated by S_2 , bound the error.

[2]

Exercises 2.1-2.3

2.1 State the alternating series error bound. [1]

2.1

- $|S - S_N| \leq b_{N+1}$ (the first omitted term)
-

2.2 For $\sum \frac{(-1)^{n+1}}{n}$ approximated by S_5 , bound the error. [2]

2.2

- Error $\leq b_6 = \frac{1}{6} \approx 0.167$
-

2.3 For $\sum \frac{(-1)^n}{n^2}$ approximated by S_2 , bound the error. [2]

2.3

- Error $\leq b_3 = \frac{1}{9} \approx 0.111$

Exercise 3.1

3.1 For a convergent alternating series, the error $|S - S_N|$ is at most

[1]

☐

A b_N

B b_{N+1}

C S_N

D $\sum b_n$

Exercise 3.1

3.1 For a convergent alternating series, the error $|S - S_N|$ is at most [1]

☐

A b_N

B b_{N+1}

C S_N

D $\sum b_n$

3.1 B

- the error is at most the first omitted term b_{N+1}

Exercise 3.2 (a)

3.2 The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ is approximated by its first three terms.

(a) Write the first omitted term.

[1]

Exercise 3.2 (a)

3.2 The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ is approximated by its first three terms.

(a) Write the first omitted term.

[1]

(a)

- $b_4 = \frac{1}{4^3} = \frac{1}{64}$

Exercise 3.2 (b)

3.2 The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ is approximated by its first three terms.

(b) Give an upper bound for the error.

[2]

Exercise 3.2 (b)

3.2 The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ is approximated by its first three terms.

(b) Give an upper bound for the error.

[2]

(b)

- $\text{Error} \leq \frac{1}{64} \approx 0.0156$

Exercise 3.3

3.3 How many terms of $\sum \frac{(-1)^{n+1}}{n}$ are needed to guarantee an error less than 0.05? [3]

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3.3

- Need $b_{N+1} = \frac{1}{N+1} < 0.05$; $N + 1 > 20$, so $N \geq 20$; take 20 terms