

Exercise walk-through

10.1

Defining Convergent and Divergent Infinite Series

AP Calculus BC

Geometric series – the one you can sum exactly

A **geometric series** has a constant ratio r . It converges **exactly** when $|r| < 1$:

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

It underlies **power series** later in the unit.

Sum $3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \dots$

Here $a = 3$ and $r = \frac{1}{2}$, and $|r| < 1$:

$$\frac{a}{1-r} = \frac{3}{1-\frac{1}{2}} = 6.$$

Absolute against conditional convergence

Converges absolutely

A series **converges absolutely** when $\sum |a_n|$ converges – and then $\sum a_n$ converges too, whatever the signs do.

Converges conditionally

It **converges conditionally** when $\sum a_n$ converges but $\sum |a_n|$ does not – the alternating harmonic series is the standard case.

Why it matters

An absolutely convergent series can be rearranged freely; a conditionally convergent one can be rearranged to give *any* sum at all.

How to test

Test $\sum |a_n|$ first. If it converges you are finished; if not, try the alternating series test on $\sum a_n$.

“Converges” is not one property. Say which kind, because the error bound and the rearrangement both depend on it.

Exam tips

- Test a series for convergence with the right tool: **geometric** ($|r| < 1$, sum $\frac{a}{1-r}$), n th-term, ratio, integral, comparison, or alternating-series test.
- A **geometric** infinite sum converges only when $|r| < 1$; otherwise it diverges.
- Build a **Taylor/Maclaurin** series to approximate a function; more terms give a better fit **near the centre**.
- Know the standard Maclaurin series for e^x , $\sin x$, $\cos x$, and $\frac{1}{1-x}$.
- Find the **radius/interval of convergence** with the ratio test, then check the endpoints separately.

Exercises 2.1-2.3

2.1 Write the third partial sum S_3 of $\sum_{n=1}^{\infty} \frac{1}{n}$. [1]

2.2 A series has partial sums $S_N = 3 - \frac{1}{N}$. State its sum. [1]

2.3 A series has $S_N = 2N$. State whether it converges or diverges. [1]

Exercises 2.1-2.3

2.1 Write the third partial sum S_3 of $\sum_{n=1}^{\infty} \frac{1}{n}$. [1]

2.1

- $S_3 = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$

2.2 A series has partial sums $S_N = 3 - \frac{1}{N}$. State its sum. [1]

2.2

- Sum = 3 (as $N \rightarrow \infty$, $\frac{1}{N} \rightarrow 0$)

2.3 A series has $S_N = 2N$. State whether it converges or diverges. [1]

2.3

- Diverges ($S_N \rightarrow \infty$)

Exercise 3.1

3.1 A series $\sum a_n$ converges if

[1]

☐

A $a_n \rightarrow 0$

B the partial sums S_N approach a finite limit

C a_n is always positive

D there are infinitely many terms

Exercise 3.1

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[1]

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A $a_n \rightarrow 0$

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3.1 B

- convergence is defined by the partial sums approaching a finite limit

Exercise 3.2 (a)

3.2 For the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$:

(a) Write S_3 .

[2]

Exercise 3.2 (a)

3.2 For the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$:

(a) Write S_3 .

[2]

(a)

- $S_3 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

Exercise 3.2 (b)

3.2 For the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$:

(b) Find a formula for S_N and hence the sum.

[3]

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(b) Find a formula for S_N and hence the sum.

[3]

(b)

- $S_N = 1 - \frac{1}{N+1}$; as $N \rightarrow \infty$, sum = 1

Exercise 3.3

3.3 A series has partial sums $S_N = \frac{N}{N+1}$. State the sum of the series, with a reason. [2]

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3.3

- $\frac{N}{N+1} \rightarrow 1$, so the series converges to 1