Exercise walk-through

1.9 Connecting Multiple Representations of Limits

AP Calculus BC

The limit theorems

If both limits exist, the limit of a combination is that combination of the limits:

$$\lim(f \pm g) = \lim f \pm \lim g$$

$$\lim(fg) = \lim f \cdot \lim g$$

$$\lim \frac{f}{g} = \frac{\lim f}{\lim g} \quad (\text{bottom} \neq 0)$$

$$\lim g(f(x)) = g(\lim f(x)) \quad (g \text{ continuous})$$

The practical rule: try **direct substitution** first. A real number out means you are done.

The vocabulary a limit question uses

Piecewise-defined function	A **piecewise-defined function** uses different rules on different intervals – so the one-sided limits at a join must be checked separately.
Composite	For a **composite** $f(g(x))$, the limit passes inside when f is continuous there: $\lim f(g(x)) = f(\lim g(x))$.
Radical	A **radical** expression with an indeterminate form usually needs the conjugate – multiply top and bottom by it and the difference of squares clears the root.
Continuous on an interval	A function is **continuous on an interval** when it is continuous at every point of it – and one-sided at the endpoints.
Existence theorem	An **existence theorem** – the Intermediate Value Theorem, the Extreme Value Theorem – guarantees that something exists without saying where.

> The **instantaneous rate of change** is the limit of an average rate as the interval shrinks. That sentence is the whole idea of the derivative.

When substitution gives $\frac{0}{0}$

$\frac{0}{0}$ is an **indeterminate form** – it does not mean the limit fails. Rewrite into an **equivalent form**, then substitute.

> The cancelled factor is why the graph had a hole: the two functions agree everywhere except at c.

Three standard moves

Factor and cancel:

$$\lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2}(x+2) = 4$$

Multiply by the **conjugate**:

$$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} = \frac{1}{2}$$

Or use a trig identity.

Choosing a procedure

1. substitute – a real answer means done

2. $\frac{0}{0}$? rewrite, then substitute

3. $\frac{5}{0}$? infinite or DNE – check each side

4. $x \to \pm\infty$? compare the **dominant** terms

5. trapped between two functions? squeeze theorem

Choosing a method for a limit

1. substitute	if it gives a number, that is the limit – for a continuous function
2. factor and cancel	for $0/0$ from a common factor
3. rationalise	multiply by the conjugate when a surd causes the $0/0$
4. use identities	alternate forms of trigonometric functions often simplify what algebra cannot
5. squeeze	trap the function between two with the same limit
6. compare degrees	for a limit at infinity, the leading terms decide

> This is a skill topic, not new content: choose the right tool for the limit in front of you. And the same limit lives in a graph, a table and an algebraic form – move between them.

Exercises 2.1·2.3

2.1 Name three ways to find a limit. [1]

2.2 State what should be true across the representations. [1]

2.3 If algebra gives 5 and the table trend is 5, state the limit. [1]

Exercises 2.1·2.3

2.1 Name three ways to find a limit. [1]

2.1

- from a graph, a table, or algebra

2.2 State what should be true across the representations. [1]

2.2

- they should all give the same value

2.3 If algebra gives 5 and the table trend is 5, state the limit. [1]

2.3

- 5

Exercises 3.1·3.2

3.1 A limit can be represented by [1]

- **A** only a graph
- **B** a graph, table, or algebra
- **C** only algebra
- **D** none of these

3.2 The three representations of a limit should [1]

- **A** conflict
- **B** all agree
- **C** give different values
- **D** be ignored

Exercises 3.1·3.2

3.1 A limit can be represented by [1]

- **A** only a graph
- **B** a graph, table, or algebra
- **C** only algebra
- **D** none of these

3.1 B

- a graph, table, or algebra.

3.2 The three representations of a limit should [1]

- **A** conflict
- **B** all agree
- **C** give different values
- **D** be ignored

3.2 B

- all agree.

Exercise 3.3 (a)

3.3 For a function, algebra gives $\lim = 4$, the table trend is $\to 4$, and the graph approaches 4.

(a) State the value from algebra. [1]

Exercise 3.3 (a)

3.3 For a function, algebra gives $\lim = 4$, the table trend is $\to 4$, and the graph approaches 4.

(a) State the value from algebra. [1]

(a)

- 4

Exercise 3.3 (b)

3.3 For a function, algebra gives $\lim = 4$, the table trend is $\to 4$, and the graph approaches 4.

(b) State whether they agree. [1]

Exercise 3.3 (b)

3.3 For a function, algebra gives $\lim = 4$, the table trend is $\to 4$, and the graph approaches 4.

(b) State whether they agree. [1]

(b)

- yes

Exercise 3.3 (c)

3.3 For a function, algebra gives $\lim = 4$, the table trend is $\to 4$, and the graph approaches 4.

(c) State the limit. [1]

Exercise 3.3 (c)

3.3 For a function, algebra gives $\lim = 4$, the table trend is $\to 4$, and the graph approaches 4.

(c) State the limit. [1]

(c)

- 4

Exercise 3.4

3.4 Let $g(x) = (x^2 - 9)/(x - 3)$ for $x \neq 3$ and $g(3) = 9$. (a) Find the limit at 3 by algebra. [2]

(b) State the open and filled point coordinates at $x = 3$ and decide whether g is continuous there. [2]

Exercise 3.4

3.4 Let $g(x) = (x^2 - 9)/(x - 3)$ for $x \neq 3$ and $g(3) = 9$. (a) Find the limit at 3 by algebra. [2]

(b) State the open and filled point coordinates at $x = 3$ and decide whether g is continuous there. [2]

3.4

(a) For $x \neq 3$, $g(x) = x + 3$, so the limit is 6.

(b) Open point $(3, 6)$ and filled point $(3, 9)$; the value differs from the limit, so the function is not continuous.