

Exercise walk-through

1.7

Selecting Procedures for Determining Limits

AP Calculus BC

The limit theorems

If both limits exist, the limit of a combination is that combination of the limits:

$$\lim(f \pm g) = \lim f \pm \lim g$$

$$\lim(fg) = \lim f \cdot \lim g$$

$$\lim \frac{f}{g} = \frac{\lim f}{\lim g} \quad (\text{bottom} \neq 0)$$

$$\lim g(f(x)) = g(\lim f(x)) \quad (g \text{ continuous})$$

The practical rule: try **direct substitution** *first*. A real number out means you are done.

The vocabulary a limit question uses

Piecewise-defined function

A **piecewise-defined function** uses different rules on different intervals – so the one-sided limits at a join must be checked separately.

Composite

For a **composite** $f(g(x))$, the limit passes inside when f is continuous there:
 $\lim f(g(x)) = f(\lim g(x))$.

Radical

A **radical** expression with an indeterminate form usually needs the conjugate – multiply top and bottom by it and the difference of squares clears the root.

Continuous on an interval

A function is **continuous on an interval** when it is continuous at every point of it – and one-sided at the endpoints.

Existence theorem

An **existence theorem** – the Intermediate Value Theorem, the Extreme Value Theorem – guarantees that something exists without saying where.

The **instantaneous rate of change** is the limit of an average rate as the interval shrinks. That sentence is the whole idea of the derivative.

When substitution gives $\frac{0}{0}$

$\frac{0}{0}$ is an **indeterminate form** –
 it does **not** mean the limit fails.
 Rewrite into an **equivalent form**, then substitute.

The cancelled factor is **why the graph had a hole**: the two functions agree everywhere except at c .

Three standard moves

Factor and cancel :

$$\lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2} (x+2) = 4$$

Multiply by the **conjugate** :

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \frac{1}{2}$$

Or use a **trig identity**.

Choosing a procedure

1. substitute – a real
answer means **done**

2. $\frac{0}{0}$? **rewrite**, then substitute

3. $\frac{\infty}{0}$? infinite or DNE
– **check each side**

4. $x \rightarrow \pm\infty$? compare
the **dominant** terms

5. trapped between two
functions? **squeeze theorem**

Choosing a method for a limit

- 1. substitute** if it gives a number, that is the limit – for a continuous function
- 2. factor and cancel** for $0/0$ from a common factor
- 3. rationalise** multiply by the conjugate when a surd causes the $0/0$
- 4. use identities** alternate forms of trigonometric functions often simplify what algebra cannot
- 5. squeeze** trap the function between two with the same limit
- 6. compare degrees** for a limit at infinity, the leading terms decide

This is a **skill** topic, not new content: choose the right tool for the limit in front of you. And the same limit lives in a graph, a table and an algebraic form – move between them.

Exercises 2.1-2.3

2.1 State the first method to try.

[1]

2.2 State what a $\frac{0}{0}$ result signals.

[1]

2.3 State what a $\frac{\text{nonzero}}{0}$ result signals.

[1]

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[1]

2.1

- direct substitution

2.2 State what a $\frac{0}{0}$ result signals.

[1]

2.2

- that more algebra (factoring/rationalizing) is needed

2.3 State what a $\frac{\text{nonzero}}{0}$ result signals.

[1]

2.3

- a possible vertical asymptote – check one-sided limits

Exercises 3.1-3.2

3.1 The first thing to try for a limit is

[1]

☐

A factoring

B direct substitution

C rationalizing

D a table

☐

3.2 A $\frac{0}{0}$ form means

[1]

A the limit is 0

B more algebra is needed

C the limit is ∞

D no limit exists

Exercises 3.1-3.2

3.1 The first thing to try for a limit is

[1]

☐

- A factoring
- B direct substitution
- C rationalizing
- D a table

3.1 B

- direct substitution.

3.2 A $\frac{0}{0}$ form means

[1]

☐

- A the limit is 0
- B more algebra is needed
- C the limit is ∞
- D no limit exists

3.2 B

- more algebra is needed.

Exercise 3.3 (a)

$$\mathbf{3.3} \lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}.$$

(a) Substitute to see the form.

[1]

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(a)

- $\frac{0}{0}$

Exercise 3.3 (b)

3.3 $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}.$

(b) Factor the denominator.

[1]

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(b)

- $(x - 2)(x + 2)$

Exercise 3.3 (c)

3.3 $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}.$

(c) Simplify and evaluate.

[1]

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3.3 $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}.$

(c) Simplify and evaluate.

[1]

(c)

- $\frac{1}{x + 2} = \frac{1}{4}$

Exercise 3.4

3.4 (a) Find $\lim_{x \rightarrow 1} (x^2 - 1)/(x - 1)$ and name your method. [2]

(b) Describe both one-sided limits of $1/(x - 1)$ at 1 and decide whether a two-sided limit exists. [2]

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3.4

(a) Factor and cancel $x - 1$ for $x \neq 1$; the limit of $x + 1$ is 2.

(b) Left: $-\infty$; right: $+\infty$; no two-sided limit exists because the two sides do not agree.