

Exercise walk-through

1.3

Estimating Limit Values from Graphs

AP Calculus BC

Defining a limit

$\lim_{x \rightarrow c} f(x) = R$ means $f(x)$ can be made
arbitrarily close to R by taking x
sufficiently close to c – but not equal
to c .

A limit describes the **behaviour near** c ,
not the value at c .

graphically

numerically

analytically

Moving between the three is a core skill.

Reading a limit from a graph

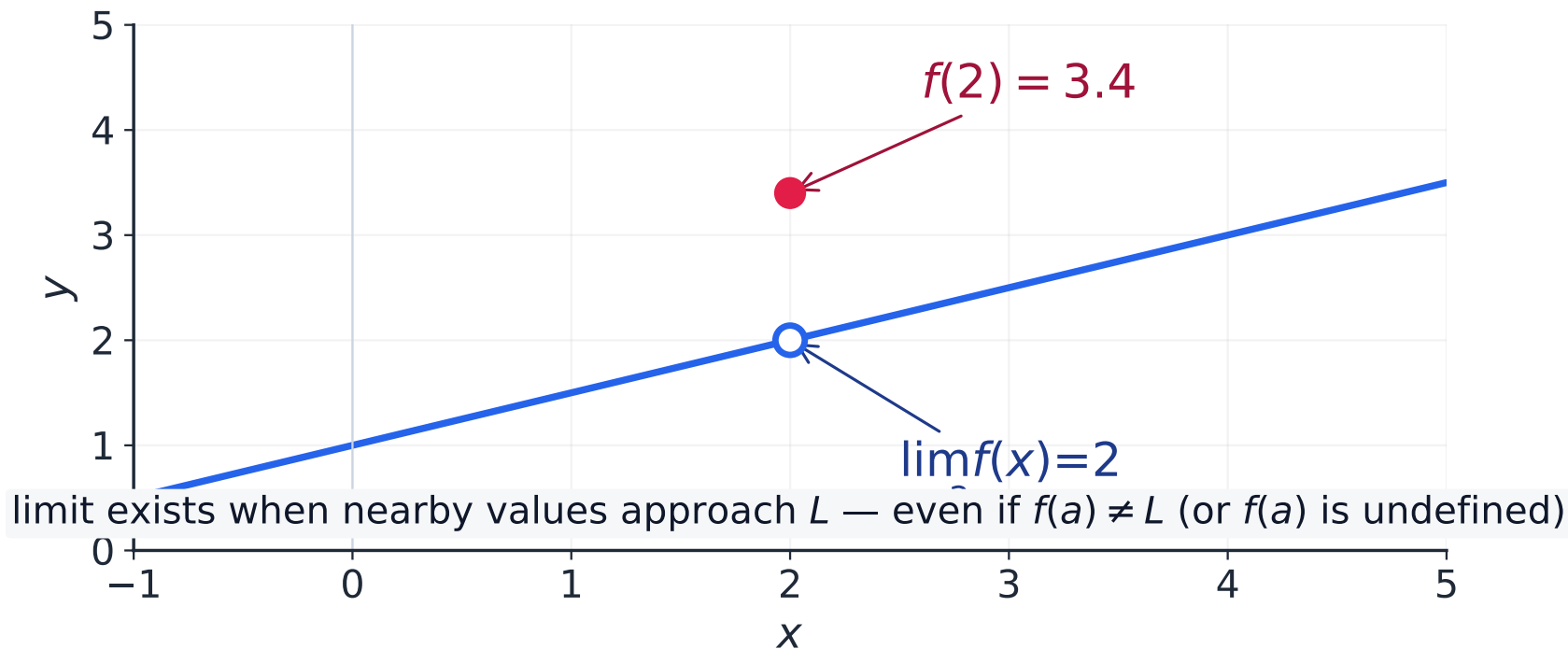
Trace toward c from each side:

- from the left: **left-hand limit**
- from the right: **right-hand limit**

If both head to the **same** height, the two-sided limit exists.

Open circle : a height approached but **not reached**. **Closed circle** : the actual value.

Reading a limit from a graph, in detail



The open circle is the height the curve approaches (the limit); the filled dot is the actual value $f(2)$ — they need not agree.

When a limit does not exist

Jump

the two sides **disagree**

$$\lim_{x \rightarrow 0} \frac{|x|}{x} \text{ DNE}$$

Unbounded

grows without limit

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$

Oscillates

forever near c

$$\lim_{x \rightarrow 0} \sin \frac{1}{x} \text{ DNE}$$

Watch the **scale** : a zoomed-out graph can hide behaviour near a point – confirm with algebra.

Estimating a limit from a table

Estimate $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$ (it is $\frac{0}{0}$ at $x = 2$)

x	1.9	1.99	1.999	$\rightarrow 2 \leftarrow$	2.001	2.01	2.1
$f(x)$	3.9	3.99	3.999	?	4.001	4.01	4.1

Both sides march toward 4.

A table only **suggests** a value – it is a numerical **estimate**, not a proof.

Exercises 2.1-2.3

2.1 State when a two-sided limit exists.

[1]

2.2 State the notation for a left-hand limit.

[1]

2.3 If the limit from the left is 4 and from the right is 4, state the two-sided limit.

[2]

Exercises 2.1-2.3

2.1 State when a two-sided limit exists. [1]

2.1

- when the left-hand and right-hand limits are equal
-

2.2 State the notation for a left-hand limit. [1]

2.2

- $\lim_{x \rightarrow a^-} f(x)$
-

2.3 If the limit from the left is 4 and from the right is 4, state the two-sided limit. [2]

2.3

- the two-sided limit is 4

Exercises 3.1·3.2

3.1 A two-sided limit exists if and only if

[1]

☐

A $f(a)$ is defined

B the left and right limits are equal

C the graph is a straight line

D the limit is 0

3.2 A hole in the graph at $x = a$ means

[1]

☐

A the limit cannot exist

B the limit may still exist

C f is linear

D $a = 0$

Exercises 3.1-3.2

3.1 A two-sided limit exists if and only if

[1]

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3.1 B

- the left and right limits are equal.

3.2 A hole in the graph at $x = a$ means

[1]

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A the limit cannot exist

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C f is linear

D $a = 0$

3.2 B

- the limit may still exist.

Exercise 3.3 (a)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(a) State the left-hand limit.

[1]

Exercise 3.3 (a)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(a) State the left-hand limit.

[1]

(a)

- 5

Exercise 3.3 (b)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(b) State the right-hand limit.

[1]

Exercise 3.3 (b)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(b) State the right-hand limit.

[1]

(b)

- 3

Exercise 3.3 (c)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(c) State whether the two-sided limit exists.

[1]

Exercise 3.3 (c)

3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(c) State whether the two-sided limit exists.

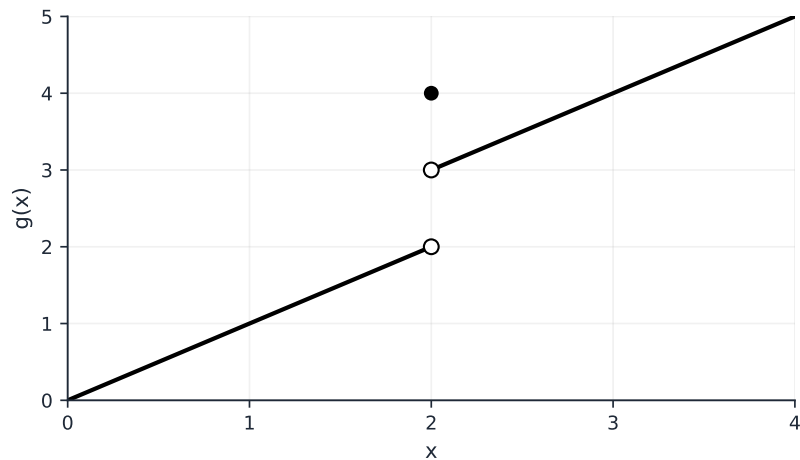
[1]

(c)

- no, the one-sided limits differ

Setup for exercise 3.4

3.4 The graph defines g near $x = 2$.



Exercise 3.4 (a)

(a) State its left- and right-hand limits at 2.

[2]

Exercise 3.4 (a)

(a) State its left- and right-hand limits at 2.

[2]

(a) The left-hand limit is 2 and the right-hand limit is 3.

Exercise 3.4 (b)

(b) Does the two-sided limit exist? Explain using the graph.

[2]

Exercise 3.4 (b)

- (b) Does the two-sided limit exist? Explain using the graph. [2]
- (b) The two-sided limit does not exist, because the one-sided approaching heights differ. The filled point at 4 does not resolve that mismatch.