

Exercise walk-through

1.15

Connecting Limits at Infinity and Horizontal Asymptotes

AP Calculus BC

Infinite limits and vertical asymptotes

Where a **non-zero number** is divided by something approaching 0, the graph hugs the line $x = c$ – a **vertical asymptote**.

Always check **each side**: the two sides can shoot **opposite** ways.

End behaviour, from the leading terms

Relative magnitudes

At infinity only the **relative magnitudes** matter: exponentials beat any polynomial, and a polynomial beats any logarithm.

Equal degrees

When numerator and denominator have the same degree, the limit is the **ratio of the leading coefficients** – and that ratio is the horizontal asymptote.

Unequal degrees

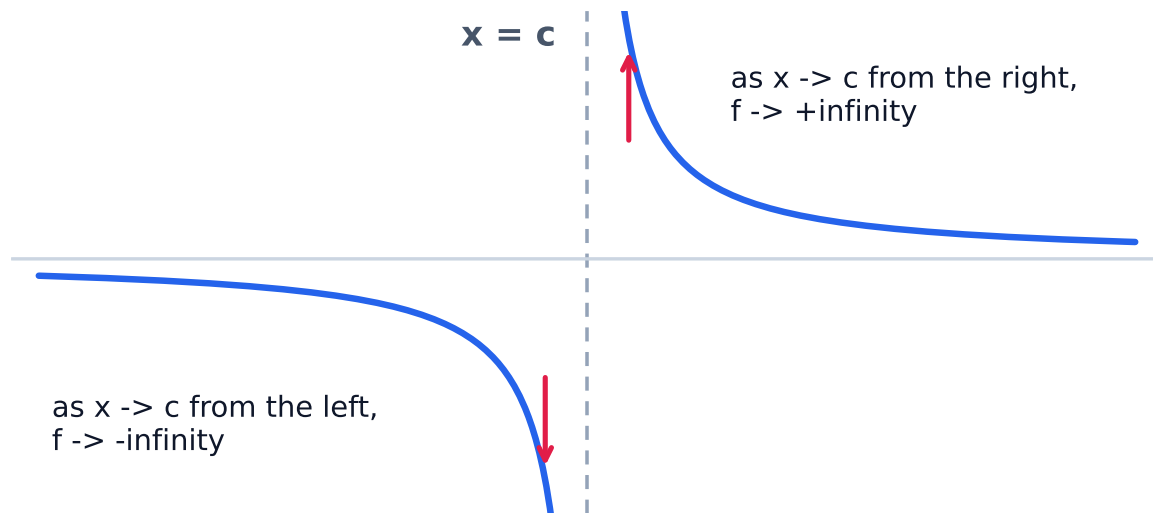
Numerator smaller: the limit is 0. Numerator larger: it grows without bound, and polynomial division gives the slant asymptote.

Naming it

That long-run **behavior** – the **end behavior** – is what a limit at infinity reports.

Divide every term by the highest power in the denominator. Every rational end-behaviour limit falls out of that one step.

Infinite limits and vertical asymptotes, in detail



vertical asymptote $x = c$ when $\lim_{x \rightarrow c^+} f(x) = \pm\infty$ or $\lim_{x \rightarrow c^-} f(x) = \pm\infty$

Where a **non-zero number** is divided by something approaching 0, the graph hugs the line $x = c$ – a **vertical asymptote**.

Exploring Types of Discontinuities

A function is **discontinuous** at c when its graph “breaks” there.

- **Removable discontinuity** – a single hole. The two-sided limit exists, but the point is missing or misplaced.
- **Jump discontinuity** – the two one-sided limits exist but disagree, so the curve jumps.
- **Infinite discontinuity asymptote** – the function blows up to $\pm\infty$ at a **vertical** .

Limits at infinity and horizontal asymptotes

Let the **input** grow: **end behaviour** as $x \rightarrow \pm\infty$. If outputs settle toward L , then $y = L$ is a **horizontal asymptote**.

Far out, an **exponential beats any polynomial**, and a polynomial beats any logarithm.

Exercises 2.1·2.3

2.1 State what $\lim_{x \rightarrow \infty} f(x) = L$ indicates. [1]

2.2 For $f(x) = \frac{1}{x}$, state $\lim_{x \rightarrow \infty} f(x)$. [1]

2.3 For $f(x) = \frac{2x}{x+1}$, state the horizontal asymptote. [2]

Exercises 2.1-2.3

2.1 State what $\lim_{x \rightarrow \infty} f(x) = L$ indicates. [1]

2.1

- the graph has a horizontal asymptote $y = L$

2.2 For $f(x) = \frac{1}{x}$, state $\lim_{x \rightarrow \infty} f(x)$. [1]

2.2

- 0

2.3 For $f(x) = \frac{2x}{x+1}$, state the horizontal asymptote. [2]

2.3

- $y = 2$ (equal degrees, ratio 2/1)

Exercises 3.1-3.2

3.1 $\lim_{x \rightarrow \infty} f(x) = L$ means $y = L$ is a

[1]

☐

- A vertical asymptote
- B horizontal asymptote
- C hole
- D root

3.2 $\lim_{x \rightarrow \infty} \frac{1}{x}$ equals

[1]

☐

- A 1
- B 0
- C ∞
- D -1

Exercises 3.1-3.2

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- A vertical asymptote
- B horizontal asymptote
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3.1 B

- horizontal asymptote.

3.2 $\lim_{x \rightarrow \infty} \frac{1}{x}$ equals

[1]

☐

- A 1
- B 0
- C ∞
- D -1

3.2 B

- 0.

Exercise 3.3 (a)

3.3 $f(x) = \frac{3x^2}{x^2 + 1}.$

(a) Compare the degrees.

[1]

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(a) Compare the degrees.

[1]

(a)

- equal (both degree 2)

Exercise 3.3 (b)

3.3 $f(x) = \frac{3x^2}{x^2 + 1}.$

(b) Take the ratio of the leading coefficients.

[1]

Exercise 3.3 (b)

3.3 $f(x) = \frac{3x^2}{x^2 + 1}.$

(b) Take the ratio of the leading coefficients.

[1]

(b)

- $3/1 = 3$

Exercise 3.3 (c)

3.3 $f(x) = \frac{3x^2}{x^2 + 1}.$

(c) State the horizontal asymptote.

[1]

Exercise 3.3 (c)

3.3 $f(x) = \frac{3x^2}{x^2 + 1}.$

(c) State the horizontal asymptote.

[1]

(c)

- $y = 3$

Exercise 3.4

3.4 (a) Find $\lim_{x \rightarrow \infty} (2x^2 - x)/(5x^2 + 1)$, showing your algebraic method. [2]

(b) State the horizontal asymptote and explain whether it determines the value at $x = 0$. [2]

Exercise 3.4

3.4 (a) Find $\lim_{x \rightarrow \infty} (2x^2 - x)/(5x^2 + 1)$, showing your algebraic method. [2]

(b) State the horizontal asymptote and explain whether it determines the value at $x = 0$. [2]

3.4

(a) Divide by x^2 : the limit is $(2 - 1/x)/(5 + 1/x^2) \rightarrow 2/5$.

(b) $y = 2/5$; it describes end behaviour, whereas direct substitution gives $g(0) = 0$.