

Exercise walk-through

1.14

Connecting Infinite Limits and Vertical Asymptotes

AP Calculus BC

Infinite limits and vertical asymptotes

Where a **non-zero number** is divided by something approaching 0, the graph hugs the line $x = c$ – a **vertical asymptote**.

Always check **each side**: the two sides can shoot **opposite** ways.

End behaviour, from the leading terms

Relative magnitudes

At infinity only the **relative magnitudes** matter: exponentials beat any polynomial, and a polynomial beats any logarithm.

Equal degrees

When numerator and denominator have the same degree, the limit is the **ratio of the leading coefficients** – and that ratio is the horizontal asymptote.

Unequal degrees

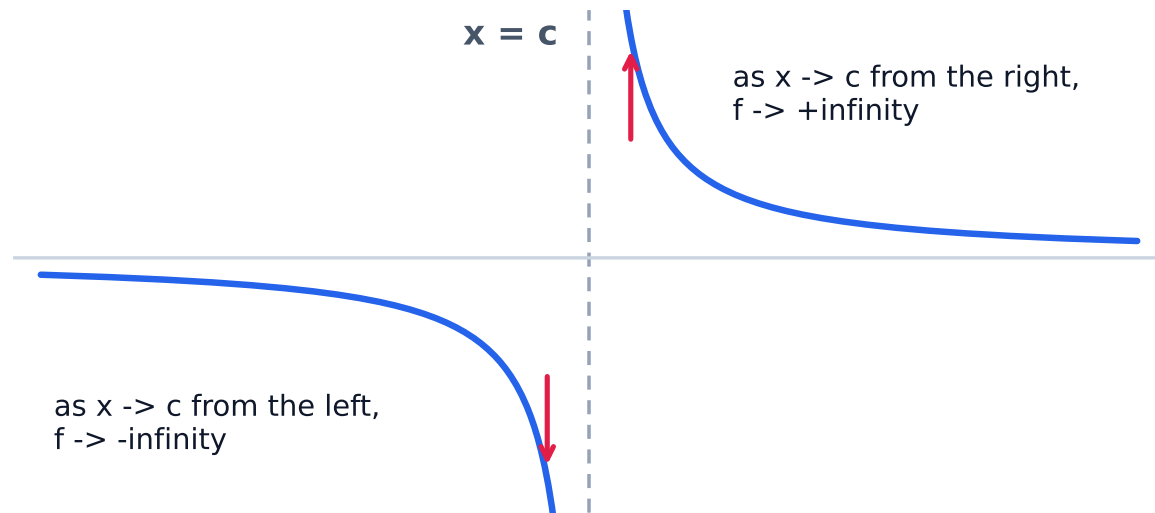
Numerator smaller: the limit is 0. Numerator larger: it grows without bound, and polynomial division gives the slant asymptote.

Naming it

That long-run **behavior** – the **end behavior** – is what a limit at infinity reports.

Divide every term by the highest power in the denominator. Every rational end-behaviour limit falls out of that one step.

Infinite limits and vertical asymptotes, in detail



vertical asymptote $x = c$ when $\lim_{x \rightarrow c^+} f(x) = \pm\infty$ or $\lim_{x \rightarrow c^-} f(x) = \pm\infty$

Where a **non-zero number** is divided by something approaching 0, the graph hugs the line $x = c$ – a **vertical asymptote**.

Exploring Types of Discontinuities

A function is **discontinuous** at c when its graph “breaks” there.

- **Removable discontinuity** – a single hole. The two-sided limit exists, but the point is missing or misplaced.
- **Jump discontinuity** – the two one-sided limits exist but disagree, so the curve jumps.
- **Infinite discontinuity asymptote** – the function blows up to $\pm\infty$ at a **vertical** .

Limits at infinity and horizontal asymptotes

Let the **input** grow: **end behaviour** as $x \rightarrow \pm\infty$. If outputs settle toward L , then $y = L$ is a **horizontal asymptote**.

Far out, an **exponential beats any polynomial**, and a polynomial beats any logarithm.

Exercises 2.1·2.3

2.1 State what an infinite limit at a indicates.

[1]

2.2 For $f(x) = \frac{1}{x}$, state the vertical asymptote.

[1]

2.3 State where a rational function has a vertical asymptote.

[2]

Exercises 2.1·2.3

2.1 State what an infinite limit at a indicates.

[1]

2.1

- the graph has a vertical asymptote there

2.2 For $f(x) = \frac{1}{x}$, state the vertical asymptote.

[1]

2.2

- $x = 0$

2.3 State where a rational function has a vertical asymptote.

[2]

2.3

- where the denominator is 0 but the numerator is not

Exercises 3.1-3.2

3.1 If $\lim_{x \rightarrow a} f(x) = \infty$, then $x = a$ is a

[1]

☐

A horizontal asymptote

B vertical asymptote

C hole

D root

3.2 $f(x) = \frac{1}{x-2}$ has a vertical asymptote at

[1]

☐

A $x = 0$

B $x = 2$

C $y = 2$

D $x = -2$

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- ☐ **A** horizontal asymptote
B vertical asymptote
C hole
D root

3.1 B

- vertical asymptote.

3.2 $f(x) = \frac{1}{x-2}$ has a vertical asymptote at

[1]

- ☐ **A** $x = 0$
B $x = 2$
C $y = 2$
D $x = -2$

3.2 B

- $x = 2$.

Exercise 3.3 (a)

3.3 $f(x) = \frac{1}{x - 5}.$

(a) Where is the denominator 0?

[1]

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(a)

- $x = 5$

Exercise 3.3 (b)

3.3 $f(x) = \frac{1}{x - 5}.$

(b) State the vertical asymptote.

[1]

Exercise 3.3 (b)

3.3 $f(x) = \frac{1}{x - 5}.$

(b) State the vertical asymptote.

[1]

(b)

- $x = 5$

Exercise 3.3 (c)

3.3 $f(x) = \frac{1}{x - 5}.$

(c) State the behaviour of $\lim_{x \rightarrow 5^+} f(x).$

[1]

Exercise 3.3 (c)

3.3 $f(x) = \frac{1}{x - 5}.$

(c) State the behaviour of $\lim_{x \rightarrow 5^+} f(x).$

[1]

(c)

- $+\infty$

Exercise 3.4

3.4 For $g(x) = (x + 1)/(x - 3)$, (a) find each one-sided limit as $x \rightarrow 3$. [2]

(b) Name the vertical asymptote and justify the signs using numerator and denominator. [2]

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(b) Name the vertical asymptote and justify the signs using numerator and denominator. [2]

3.4

(a) Left $-\infty$, right $+\infty$.

(b) The asymptote is $x = 3$; the numerator approaches positive 4 while the denominator changes from small negative to small positive.