

Exercise walk-through

1.11

Defining Continuity at a Point

AP Calculus BC

Three types of discontinuity

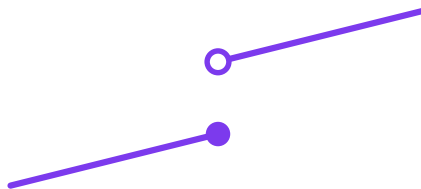
A function is **discontinuous**

at c when its graph **breaks** there:

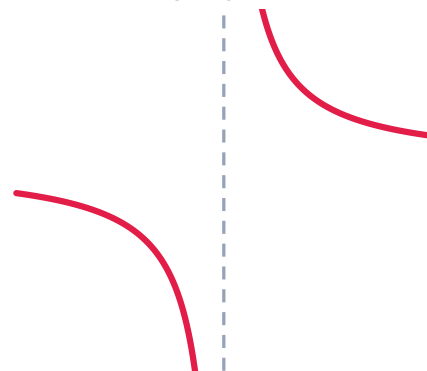
Removable
(hole)



Jump



Infinite
(asymptote)



f continuous at a iff $\lim_{x \rightarrow a} f(x) = f(a)$ (limit exists and equals the value)

Removable
(an asymptote).

(a hole) ■ **jump**

(sides disagree) ■ **infinite**

Continuity at a point – the three-part test

f is **continuous** at $x = c$ exactly when **all three** hold:

$f(c)$ **exists**

$\lim_{x \rightarrow c} f(x)$ **exists**

they **agree**: $\lim_{x \rightarrow c} f(x) = f(c)$

The point is there, the limit is there, and the two **agree**. If any one fails, f is discontinuous.

Learn it as a **checklist** – it is the backbone of nearly every continuity question.

Continuity over an interval, and repairing a hole

You rarely check point by point – whole families are continuous **on their domains**:

Polynomial, rational, power, **exponential**, **logarithmic** and **trigonometric** functions are continuous at every point of their domains.

So a rational function is continuous everywhere **except** where its denominator is zero.

Removing a discontinuity

If the limit exists at a hole, **redefine** the function there to equal the limit – the graph is repaired.

For a **piecewise** function at a boundary, solve for the **parameter** that makes the pieces **meet**:

$$\lim_{x \rightarrow c^-} f = \lim_{x \rightarrow c^+} f = f(c).$$

Exercises 2.1-2.3

2.1 State the three conditions for continuity at a . [1]

2.2 State the single equation summarizing continuity at a . [1]

2.3 If $\lim_{x \rightarrow 2} f(x) = 5$ and $f(2) = 5$, is f continuous at 2? [2]

Exercises 2.1·2.3

2.1 State the three conditions for continuity at a . [1]

2.1

- $f(a)$ is defined; the limit exists; the limit equals $f(a)$
-

2.2 State the single equation summarizing continuity at a . [1]

2.2

- $\lim_{x \rightarrow a} f(x) = f(a)$
-

2.3 If $\lim_{x \rightarrow 2} f(x) = 5$ and $f(2) = 5$, is f continuous at 2? [2]

2.3

- yes – the limit exists, $f(2)$ is defined, and they are equal

Exercises 3.1-3.2

3.1 f is continuous at a if and only if

[1]

☐

A $f(a)$ is defined

B $\lim_{x \rightarrow a} f(x) = f(a)$

C the limit exists only

D f is linear

☐

3.2 If $\lim_{x \rightarrow a} f(x)$ exists but $\neq f(a)$, then f is

[1]

A continuous at a

B discontinuous at a

C linear

D undefined everywhere

Exercises 3.1·3.2

3.1 f is continuous at a if and only if

[1]

☐

A $f(a)$ is defined

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C the limit exists only

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3.1 B

- $\lim_{x \rightarrow a} f(x) = f(a)$.

3.2 If $\lim_{x \rightarrow a} f(x)$ exists but $\neq f(a)$, then f is

[1]

☐

A continuous at a

B discontinuous at a

C linear

D undefined everywhere

3.2 B

- discontinuous at a .

Exercise 3.3 (a)

3.3 $f(2) = 5$ and $\lim_{x \rightarrow 2} f(x) = 5$.

(a) Is $f(2)$ defined?

[1]

Exercise 3.3 (a)

3.3 $f(2) = 5$ and $\lim_{x \rightarrow 2} f(x) = 5$.

(a) Is $f(2)$ defined?

[1]

(a)

- yes

Exercise 3.3 (b)

3.3 $f(2) = 5$ and $\lim_{x \rightarrow 2} f(x) = 5$.

(b) Does the limit exist?

[1]

Exercise 3.3 (b)

3.3 $f(2) = 5$ and $\lim_{x \rightarrow 2} f(x) = 5$.

(b) Does the limit exist?

[1]

(b)

- yes

Exercise 3.3 (c)

3.3 $f(2) = 5$ and $\lim_{x \rightarrow 2} f(x) = 5$.

(c) Is f continuous at 2?

[1]

Exercise 3.3 (c)

3.3 $f(2) = 5$ and $\lim_{x \rightarrow 2} f(x) = 5$.

(c) Is f continuous at 2?

[1]

(c)

- yes

Exercise 3.4

3.4 Let $g(x) = x^2 - x + 1$ for $x \neq 2$ and $g(2) = k$. (a) Find the limit at 2 and the value of k required for continuity. [2]

(b) State the three continuity conditions and explain why your assignment meets them. [2]

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3.4 Let $g(x) = x^2 - x + 1$ for $x \neq 2$ and $g(2) = k$. (a) Find the limit at 2 and the value of k required for continuity. [2]

(b) State the three continuity conditions and explain why your assignment meets them. [2]

3.4

(a) Limit $= 4 - 2 + 1 = 3$; $k = 3$.

(b) The point value and limit both exist and are equal at 3.