

Exercise walk-through

1.10

Exploring Types of Discontinuities

AP Calculus BC

What calculus is for, and the words limits are described in

calculus	the mathematics of change and of accumulation
the two questions	how fast is something changing at an instant , and how much accumulates over an interval
a limit	the value $f(x)$ approaches as x approaches a – and it ignores the point itself
the graph's distinction	a hole means the limit exists and the value does not; a filled dot elsewhere means they differ
average rate of change	the slope of a secant; the derivative is the limit of it
not tested	the epsilon–delta definition is not on the AP exam, so it is not used here

“Ignore the point itself” is the whole idea. A limit describes the **approach**, and a function can be undefined exactly where its limit is clearest.

When substitution gives $\frac{0}{0}$

$\frac{0}{0}$ is an **indeterminate form** –
 it does **not** mean the limit fails.
 Rewrite into an **equivalent form**, then substitute.

The cancelled factor is **why the graph had a hole**: the two functions agree everywhere except at c .

Three standard moves

Factor and cancel :

$$\lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2} (x+2) = 4$$

Multiply by the **conjugate** :

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \frac{1}{2}$$

Or use a **trig identity**.

Choosing a method for a limit

- 1. substitute** if it gives a number, that is the limit – for a continuous function
- 2. factor and cancel** for $0/0$ from a common factor
- 3. rationalise** multiply by the conjugate when a surd causes the $0/0$
- 4. use identities** alternate forms of trigonometric functions often simplify what algebra cannot
- 5. squeeze** trap the function between two with the same limit
- 6. compare degrees** for a limit at infinity, the leading terms decide

This is a **skill** topic, not new content: choose the right tool for the limit in front of you. And the same limit lives in a graph, a table and an algebraic form – move between them.

Exercises 2.1-2.3

2.1 Name the three types of discontinuity.

[1]

2.2 State which type is a “hole”.

[1]

2.3 State which type occurs at a vertical asymptote.

[2]

Exercises 2.1-2.3

2.1 Name the three types of discontinuity. [1]

2.1

- removable, jump, and infinite
-

2.2 State which type is a “hole”. [1]

2.2

- removable
-

2.3 State which type occurs at a vertical asymptote. [2]

2.3

- infinite

Exercises 3.1-3.2

3.1 A discontinuity where the left and right limits differ is a

[1]

☐

A removable discontinuity

B jump discontinuity

C infinite discontinuity

D point of continuity

3.2 A hole in a graph is a

[1]

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Exercises 3.1-3.2

3.1 A discontinuity where the left and right limits differ is a

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3.1 B

- jump discontinuity.

3.2 A hole in a graph is a

[1]

☐

A jump discontinuity

B removable discontinuity

C infinite discontinuity

D point of continuity

3.2 B

- removable discontinuity.

Exercise 3.3 (a)

3.3 Classify each.

(a) A vertical asymptote.

[1]

Exercise 3.3 (a)

3.3 Classify each.

(a) A vertical asymptote.

[1]

(a)

- infinite

Exercise 3.3 (b)

3.3 Classify each.

(b) A single missing point.

[1]

Exercise 3.3 (b)

3.3 Classify each.

(b) A single missing point.

[1]

(b)

- removable

Exercise 3.3 (c)

3.3 Classify each.

(c) A step where the graph jumps.

[1]

Exercise 3.3 (c)

3.3 Classify each.

(c) A step where the graph jumps.

[1]

(c)

- jump

Exercise 3.4

3.4 (a) For $f(x) = (x^2 - 4)/(x - 2)$, identify the discontinuity at 2 and give a point value that removes it. [2]

(b) A function approaches 1 from the left of 3 and 4 from the right. Classify the discontinuity and explain whether changing $f(3)$ can remove it. [2]

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3.4

(a) Removable; define $f(2) = 4$.

(b) Jump discontinuity; no single point value can make the unequal one-sided limits agree.