

Exercise walk-through

1.1

Introducing Calculus: Can Change Occur at an Instant?

AP Calculus BC

Can change occur at an instant?

A speedometer reads 60 km/h. But **at a single instant** no time passes and no distance is covered:

$$\text{speed} = \frac{0}{0} - \text{undefined.}$$

The clever move: do **not** plug in zero. Watch what the average rate **approaches** as the interval shrinks.

That approaching value is a **limit** – it powers the **derivative** (Unit 2) and, in reverse, the **integral** (Unit 6).

What calculus is for, and the words limits are described in

calculus	the mathematics of change and of accumulation
the two questions	how fast is something changing at an instant , and how much accumulates over an interval
a limit	the value $f(x)$ approaches as x approaches a – and it ignores the point itself
the graph's distinction	a hole means the limit exists and the value does not; a filled dot elsewhere means they differ
average rate of change	the slope of a secant; the derivative is the limit of it
not tested	the epsilon–delta definition is not on the AP exam, so it is not used here

“Ignore the point itself” is the whole idea. A limit describes the **approach**, and a function can be undefined exactly where its limit is clearest.

Exercises 2.1-2.3

2.1 State the average rate of change of f on $[a, b]$. [1]

2.2 State what the instantaneous rate of change is the limit of. [1]

2.3 For $f(x) = x^2$, find the average rate of change on $[1, 3]$. [2]

Exercises 2.1·2.3

2.1 State the average rate of change of f on $[a, b]$. [1]

2.1

- $\frac{f(b) - f(a)}{b - a}$

2.2 State what the instantaneous rate of change is the limit of. [1]

2.2

- the average rates of change over intervals shrinking to zero

2.3 For $f(x) = x^2$, find the average rate of change on $[1, 3]$. [2]

2.3

- $\frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = 4$

Exercises 3.1·3.2

3.1 The instantaneous rate of change is

[1]

☐

A the average over $[a, b]$

B the limit of average rates as the interval shrinks

C always zero

D the slope of a single secant line

3.2 The average rate of change of f on $[a, b]$ is

[1]

☐

A $f(b) \cdot f(a)$

B $\frac{f(b) - f(a)}{b - a}$

C $f(b) + f(a)$

D $b - a$

Exercises 3.1·3.2

3.1 The instantaneous rate of change is

[1]

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A the average over $[a, b]$

B the limit of average rates as the interval shrinks

C always zero

D the slope of a single secant line

3.1 B

- the limit of average rates as the interval shrinks.

3.2 The average rate of change of f on $[a, b]$ is

[1]

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A $f(b) \cdot f(a)$

B $\frac{f(b) - f(a)}{b - a}$

C $f(b) + f(a)$

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3.2 B

- $\frac{f(b) - f(a)}{b - a}$.

Exercise 3.3 (a)

3.3 $f(x) = x^2$.

(a) Find $f(1)$ and $f(3)$.

[1]

Exercise 3.3 (a)

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(a) Find $f(1)$ and $f(3)$.

[1]

(a)

- $f(1) = 1, f(3) = 9$

Exercise 3.3 (b)

3.3 $f(x) = x^2$.

(b) Find the average rate of change on $[1, 3]$.

[1]

Exercise 3.3 (b)

3.3 $f(x) = x^2$.

(b) Find the average rate of change on $[1, 3]$.

[1]

(b)

- $\frac{9 - 1}{2} = 4$

Exercise 3.3 (c)

3.3 $f(x) = x^2$.

(c) State what a shrinking interval approaches.

[1]

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(c) State what a shrinking interval approaches.

[1]

(c)

- the instantaneous rate of change

Exercise 3.4

3.4 A position is $s(t) = t^2 + 2t$ metres. (a) Find the average velocity on $[1, 2]$. [2]

(b) Simplify the average-velocity quotient on $[1, 1+h]$ and use it to find the instantaneous velocity at $t = 1$. [2]

Exercise 3.4

3.4 A position is $s(t) = t^2 + 2t$ metres. (a) Find the average velocity on $[1, 2]$. [2]

(b) Simplify the average-velocity quotient on $[1, 1+h]$ and use it to find the instantaneous velocity at $t = 1$. [2]

3.4

(a) $v_{\text{avg}} = [s(2) - s(1)]/(2 - 1) = (8 - 3)/1 = 5 \text{ m s}^{-1}$.

(b) $v(h) = [(1+h)^2 + 2(1+h) - 3]/h = 4 + h$ for $h \neq 0$; $\lim_{h \rightarrow 0} v(h) = 4 \text{ m s}^{-1}$.