

5. Two particles,  $H$  and  $J$ , are moving along the  $x$ -axis. For  $0 \leq t \leq 5$ , the position of particle  $H$  at time  $t$  is given by  $x_H(t) = e^{t^2 - 4t}$  and the velocity of particle  $J$  at time  $t$  is given by  $v_J(t) = 2t(t^2 - 1)^3$ .
- A. Find the velocity of particle  $H$  at time  $t = 1$ . Show the work that leads to your answer.
- B. During what open intervals of time  $t$ , for  $0 < t < 5$ , are particles  $H$  and  $J$  moving in opposite directions? Give a reason for your answer.
- C. It can be shown that  $v_J'(2) > 0$ . Is the speed of particle  $J$  increasing, decreasing, or neither at time  $t = 2$ ? Give a reason for your answer.
- D. Particle  $J$  is at position  $x = 7$  at time  $t = 0$ . Find the position of particle  $J$  at time  $t = 2$ . Show the work that leads to your answer.

5. Consider the curve defined by the equation  $x^2 + 3y + 2y^2 = 48$ . It can be shown that  $\frac{dy}{dx} = \frac{-2x}{3 + 4y}$ .
- (a) There is a point on the curve near  $(2, 4)$  with  $x$ -coordinate 3. Use the line tangent to the curve at  $(2, 4)$  to approximate the  $y$ -coordinate of this point.
- (b) Is the horizontal line  $y = 1$  tangent to the curve? Give a reason for your answer.
- (c) The curve intersects the positive  $x$ -axis at the point  $(\sqrt{48}, 0)$ . Is the line tangent to the curve at this point vertical? Give a reason for your answer.
- (d) For time  $t \geq 0$ , a particle is moving along another curve defined by the equation  $y^3 + 2xy = 24$ . At the instant the particle is at the point  $(4, 2)$ , the  $y$ -coordinate of the particle's position is decreasing at a rate of 2 units per second. At that instant, what is the rate of change of the  $x$ -coordinate of the particle's position with respect to time?

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**Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.**

|                                |   |     |      |     |      |     |
|--------------------------------|---|-----|------|-----|------|-----|
| $t$<br>(seconds)               | 0 | 60  | 90   | 120 | 135  | 150 |
| $f(t)$<br>(gallons per second) | 0 | 0.1 | 0.15 | 0.1 | 0.05 | 0   |

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function  $f$ , where  $f(t)$  is measured in gallons per second and  $t$  is measured in seconds since pumping began. Selected values of  $f(t)$  are given in the table.

- (a) Using correct units, interpret the meaning of  $\int_{60}^{135} f(t) dt$  in the context of the problem. Use a right

Riemann sum with the three subintervals  $[60, 90]$ ,  $[90, 120]$ , and  $[120, 135]$  to approximate the value of

$$\int_{60}^{135} f(t) dt.$$

- (b) Must there exist a value of  $c$ , for  $60 < c < 120$ , such that  $f'(c) = 0$ ? Justify your answer.

- (c) The rate of flow of gasoline, in gallons per second, can also be modeled by  $g(t) = \left(\frac{t}{500}\right) \cos\left(\left(\frac{t}{120}\right)^2\right)$  for

$0 \leq t \leq 150$ . Using this model, find the average rate of flow of gasoline over the time interval  $0 \leq t \leq 150$ .

Show the setup for your calculations.

- (d) Using the model  $g$  defined in part (c), find the value of  $g'(140)$ . Interpret the meaning of your answer in the context of the problem.

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