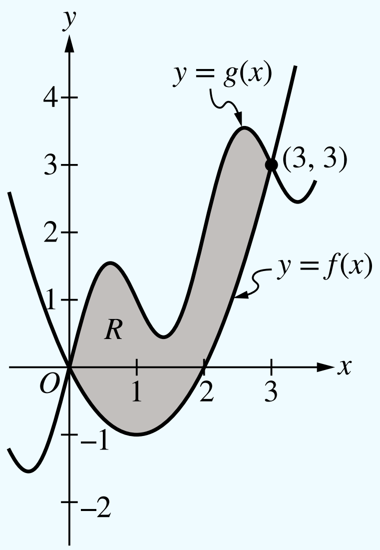
Part A (AB): Graphing calculator required

Question 2 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The shaded region R is bounded by the graphs of the functions f and g, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.



(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring	
A	Find the area of R. Show the setup for your calculations.		
	$\int_0^3 (g(x) - f(x))\,dx$	Form of integrand	**Point 1 (P1)**
	$= 5.136620$ The area is 5.137 (or 5.136).	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned for a response that presents an integrand of $g(x) - f(x)$, $|g(x) - f(x)|$, $f(x) - g(x)$, or $|f(x) - g(x)|$ in a definite integral, with or without the differential dx.
- **P1** could also be earned for a difference of definite integrals with integrands $g(x)$ and $f(x)$.
- **P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect communication between the integral and the correct answer is treated as scratch work and is not considered in scoring.
  - $\int_0^3 (f(x) - g(x))\,dx = -5.137$ so the area is 5.137.

    Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer.
  - $\int_0^3 (f(x) - g(x))\,dx = 5.137$

    Note: This response earns **P1** for the integral. It also earns **P2** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- The exact answer is $\dfrac{4 + 9\pi}{2\pi}$.

B	Region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a rectangle with height x and base in region R. Find the volume of the solid. Show the setup for your calculations.		
	$\int_0^3 x(g(x) - f(x))\,dx$	Form of integrand	**Point 3 (P3)**
	$= 7.704930$ The volume of the solid is 7.705 (or 7.704).	Answer	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned for a definite integral with an integrand presented as a product of two nonconstant factors, with one of the factors equal to x, $g(x) - f(x)$, or $f(x) - g(x)$.
- The presence or absence of the differential dx will not be considered in scoring **P3** or **P4**.
- **P4** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- Incorrect or unclear communication between the integral and the correct answer is treated as scratch work and is not considered in scoring. For example:
  - $\int_0^3 x(f(x) - g(x))\,dx = -7.705$ so the volume is 7.705.

    Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer.
  - $\int_0^3 x(f(x) - g(x))\,dx = 7.705$

    Note: This response earns **P3** for the integral. It also earns **P4** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)
- The exact answer is $\dfrac{12 + 27\pi}{4\pi}$.

C	Write, but do not evaluate, an integral expression for the volume of the solid generated when the region R is rotated about the horizontal line $y = -2$.		
	$\text{Volume} = \pi\int_0^3 \left((g(x)-(-2))^2 - (f(x)-(-2))^2\right) dx$	Form of integrand	**Point 5 (P5)**
		Integrand	**Point 6 (P6)**
		Limits, constant, and differential	**Point 7 (P7)**

Scoring Notes for Part C

- **P5** is earned for a definite integral with an integrand of $R^2 - r^2$ or $\left|R^2 - r^2\right|$, where one of $\{R, r\}$ is correct or a difference between g and a nonzero constant, and the other is correct or a difference between f and a nonzero constant.
- **P6** is earned for the integral $\int_0^3 \left((g(x)+2)^2 - (f(x)+2)^2\right) dx$, $\int_0^3 \left((f(x)+2)^2 - (g(x)+2)^2\right) dx$, or a mathematically equivalent expression.
- Note **P5** and **P6** could be earned for a difference of definite integrals.
- A response that presents an integral expression that does not include the constant π is eligible for **P5** and **P6** but does not earn **P7**.
- To be eligible for **P7**, a response must have earned **P5**.
- A response that reverses the difference of squares must resolve the reversal with either the constant or the limits of integration AND include the differential to earn **P7**. For example:
  - A response of $-\pi\int_0^3 \left((f(x)+2)^2 - (g(x)+2)^2\right) dx$ or $\pi\int_3^0 \left((f(x)+2)^2 - (g(x)+2)^2\right) dx$ earns **P5**, **P6**, and **P7**.
  - A response of $\pi\int_0^3 \left((f(x)+2)^2 - (g(x)+2)^2\right) dx$ earns **P5** and **P6** but does not earn **P7**.
- A response of only $\pi\int_0^3 \left((g(x)-(-2))^2 - (f(x)-(-2))^2\right) dx$ earns **P5**, **P6**, and **P7**.

D	It can be shown that $g'(x) = 1 + \pi\cos(\pi x)$. Find the value of x, for $0 < x < 1$, at which the line tangent to the graph of f is parallel to the line tangent to the graph of g.		
	$f'(x) = g'(x) \Rightarrow 2x - 2 = 1 + \pi\cos(\pi x)$	$f'(x) = g'(x)$	**Point 8 (P8)**
	$\Rightarrow x = 0.675819$ The lines tangent to the graphs of f and g are parallel at $x = 0.676$ (or 0.675).	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P8** is earned for a general statement, such as $f'(x) = g'(x)$, or for any correct equation formed by substituting $2x - 2$ for $f'(x)$, $1 + \pi\cos(\pi x)$ for $g'(x)$, or both.
- **P9** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

Part B (AB or BC): Graphing calculator not allowed

Question 3 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function R, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

t (minutes)	0	2	8	10
$R(t)$ (words per minute)	90	100	150	162

	Model Solution	Scoring	
A	Approximate $R'(1)$ using the average rate of change of R over the interval $0 \le t \le 2$. Show the work that leads to your answer. Indicate units of measure.		
	$R'(1) \approx \dfrac{R(2) - R(0)}{2 - 0}$ $= \dfrac{100 - 90}{2} = \dfrac{10}{2} = 5$ words per minute per minute	Answer with setup	**Point 1 (P1)**
		Units	**Point 2 (P2)**

Scoring Notes for Part A

- To earn **P1**, a response must present the answer along with the supporting work of a difference and a quotient using values from the table.
  - $\dfrac{100-90}{2-0}$, $\dfrac{10}{2-0}$, $\dfrac{100-90}{2}$, or $\dfrac{R(2)-R(0)}{2-0} = 5$ is sufficient to earn **P1**.
  - $\dfrac{R(2)-R(0)}{2-0}$ by itself is not sufficient to earn **P1**.
- **P2** is earned for correct units, whether or not they are attached to a numerical value for the average rate of change.
- **P2** is also earned for the units "$\text{words}/\text{minute}^2$."

B	Must there be a value c, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.		
	R is differentiable implies R is continuous.	Differentiable implies continuous	**Point 3 (P3)**
	$R(0) = 90 < 155 < R(10) = 162$ Therefore, by the Intermediate Value Theorem, there must be a value c, with $0 < c < 10$, such that $R(c) = 155$.	Answer with justification	**Point 4 (P4)**

Scoring Notes for Part B

- To earn **P3**, a response must state that R is continuous because R is differentiable (or equivalent). A response that simply states "R is continuous" without justification does not earn **P3**.
- A response does not need to earn **P3** to be eligible for **P4**.
- To earn **P4**, a response must indicate that $R(0) < 155$ (or $R(2) < 155$ or $R(8) < 155$) and $R(10) > 155$, state that "R is continuous," and answer "yes" in some way.
- To earn **P4**, a response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

C	Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)\,dt$. Show the work that leads to your answer.	
	$\int_0^{10} R(t)\,dt \approx \frac{R(0)+R(2)}{2}(2-0) + \frac{R(2)+R(8)}{2}(8-2) + \frac{R(8)+R(10)}{2}(10-8)$	Form of trapezoidal sum **Point 5 (P5)**
	$= \frac{90+100}{2}(2-0) + \frac{100+150}{2}(8-2) + \frac{150+162}{2}(10-8)$ $= \frac{190}{2}(2) + \frac{250}{2}(6) + \frac{312}{2}(2) = 190 + 750 + 312 = 1252$	Answer with supporting work **Point 6 (P6)**

Scoring Notes for Part C

- Read "=" as "≈" for **P5**.
- The form of a trapezoidal sum includes three terms, each of which includes a product of two factors, where one of the factors incorporates the $\frac{1}{2}$ as part of the product. To earn **P5**, at least five of the six factors must be correct. If any of the six factors is incorrect, the response does not earn **P6**. Consider the following examples:
  - $\frac{90+100}{2}(2-0) + \frac{100+150}{2}(8-2) + \frac{150+162}{2}(10-8)$ earns P5 and is sufficient to earn **P6**.
  - $\frac{190}{2}(2) + \frac{250}{2}(6) + \frac{312}{2}(2)$ earns P5 and is sufficient to earn **P6**.
  - $\frac{1}{2}((R(0)+R(2))(2) + (R(2)+R(8))(6) + (R(8)+R(10))(2))$ earns **P5** and is eligible for **P6**.
  - $\frac{90+100}{2}(2) + \frac{100+150}{2}(2) + \frac{150+162}{2}(2)$ earns P5 but is not eligible for **P6**.

    (Note that the factor of 2 in the second term of this expression is incorrect.)
- **Special case:** A response of $(90+100)+(100+150)3+(150+162)$ earns both **P5** and **P6**.
- To be eligible for **P6**, a response must have earned **P5**.

  Special case: A response of $95 \cdot 2 + 125 \cdot 6 + 156 \cdot 2$ earns **P6** but does not earn **P5**.
- A response of $\frac{90+100}{2}(2-0) + \frac{100+150}{2}(8-2) + \frac{150+162}{2}(10-8)$ or equivalent banks **P6** (i.e., subsequent errors in simplification will not be considered in scoring for **P6**).
- A response of $\frac{(90 \cdot 2 + 100 \cdot 6 + 150 \cdot 2) + (100 \cdot 2 + 150 \cdot 6 + 162 \cdot 2)}{2}$ or equivalent earns both **P5** and **P6**. (Note that the average of the left Riemann sum and right Riemann sum is equivalent to the trapezoidal sum.)
- A completely correct left Riemann sum (e.g., $90 \cdot 2 + 100 \cdot 6 + 150 \cdot 2 = 1080$) or a completely correct right Riemann sum (e.g., $100 \cdot 2 + 150 \cdot 6 + 162 \cdot 2 = 1424$) earns **P5** but does not earn **P6**.

D	A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function W defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.		
	$\int_0^{10} W(t)\,dt = \int_0^{10}\left(-\frac{3}{10}t^2 + 8t + 100\right)dt$	Integrand	**Point 7 (P7)**
	$= \left(-\frac{1}{10}t^3 + 4t^2 + 100t\right)\Big\|_0^{10}$	Antiderivative	**Point 8 (P8)**
	$= \left(-\frac{1}{10}\cdot 1000 + 4\cdot 100 + 100\cdot 10\right) - \left(-\frac{1}{10}\cdot 0 + 4\cdot 0 + 100\cdot 0\right)$ $= 1300$ Based on the model, the teacher has read 1300 words by the end of the 10 minutes.	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for an indefinite or definite integral with integrand $W(t)$, with or without the differential dt.
- **P8** is earned for the correct antiderivative, with or without the constant of integration.
- To be eligible for **P9**, a response must have earned **P8**.
- A response of $\left(-\frac{1}{10}\cdot 1000 + 4\cdot 100 + 100\cdot 10\right) - \left(-\frac{1}{10}\cdot 0 + 4\cdot 0 + 100\cdot 0\right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

Part A (AB or BC): Graphing calculator required
Question 1 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

Model Solution	Scoring	

(a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$. Show the work that leads to your answer and include units of measure.

$C'(5) \approx \dfrac{C(7) - C(3)}{7 - 3} = \dfrac{69 - 85}{4} = -4$ degrees Celsius per minute	Estimate with supporting work	**1 point**
	Units	**1 point**

Scoring notes:

- To earn the first point a response must include a difference and a quotient as the supporting work.
- $\dfrac{-16}{7-3}$, $\dfrac{69-85}{7-3}$, or $\dfrac{69-85}{4}$ is sufficient to earn the first point.
- A response that presents only units without a numerical approximation for $C'(5)$ does not earn the second point.
- The second point is also earned for "degrees per minute" attached to a numerical value.

Total for part (a) 2 points

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

$\int_0^{12} C(t)\,dt \approx (3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ $= 3\cdot 100 + 4\cdot 85 + 5\cdot 69 = 985$	Form of left Riemann sum	**1 point**
	Estimate	**1 point**
$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	**1 point**

Scoring notes:

- Read "=" as "≈" for the first point.
- To earn the first point at least five of the six factors in the Riemann sum must be correct. If any of the six factors is incorrect, the response does not earn the second point.
- A response of $(3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ earns the first point. Values must be pulled from the table to earn the second point.
- A response of $3\cdot 100 + 4\cdot 85 + 5\cdot 69$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A completely correct right Riemann sum (e.g., $3\cdot 85 + 4\cdot 69 + 5\cdot 55$) earns 1 of the first 2 points. An unsupported answer of 806 does not earn either of the first 2 points.
- Units will not affect scoring for the second point.
- To earn the third point the interpretation must include both "average temperature" and the time interval. The response need not include a reference to units. However, if incorrect units are given in the interpretation, the response does not earn the third point.

Total for part (b) 3 points

(c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

$C(20) = C(12) + \int_{12}^{20} C'(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 55 - 14.670812 = 40.329188$ The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.	Answer	**1 point**

Scoring notes:

- The first point is earned for a definite integral with integrand $C'(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A linkage error such as $C(20) = \int_{12}^{20} C'(t)\,dt = 55 - 14.670812$ or $\int_{12}^{20} C'(t)\,dt = -14.670812 = 40.329188$ earns the first 2 points but does not earn the third point.
- Missing differential (dt):
  - Unambiguous responses of $C(20) = C(12) + \int_{12}^{20} C'(t)$ or $C(20) = 55 + \int_{12}^{20} C'(t)$ earn the first 2 points and are eligible for the third point.
  - Ambiguous responses of $C(20) = \int_{12}^{20} C'(t) + C(12)$ or $C(20) = \int_{12}^{20} C'(t) + 55$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, these responses do not earn the third point.
- The second point is earned for adding $C(12)$ or 55 to a definite integral with a lower limit of 12, either symbolically or numerically.
- The third point is earned for an answer of $55 - 14.671$ or $-14.671 + 55$ with no additional simplification, provided there is some supporting work for these values.
- An answer of just 40.329 with no supporting work does not earn any points.

Total for part (c) 3 points

(d) For the model defined in part (c), it can be shown that $C''(t) = \dfrac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

Because $C''(t) > 0$ on the interval $12 < t < 20$, the rate of change in the temperature of the coffee, $C'(t)$, is increasing on this interval. That is, on the interval $12 < t < 20$, the temperature of the coffee is changing at an increasing rate.	Answer with reason	**1 point**

Scoring notes:

- This point is earned only for a correct answer with a correct reason that references the sign of the second derivative of C.
- A response that provides a reason based on the evaluation of $C''(t)$ at a single point does not earn this point.
- A response that uses ambiguous pronouns (such as “It is positive, so increasing”) does not earn this point.
- A response does not need to reference the interval $12 < t < 20$ to earn the point.

Total for part (d) 1 point

Total for question 1 9 points