

4. Contextual Applications of Differentiation

Starter Quiz · 课前小测

AP Calculus AB

Name: _____ Score: _____

1. $V(t)$ is water volume in litres at time t minutes. The units of $V'(t)$ are...

- ☐ litres
- ☐ litres per minute
- ☐ minutes
- ☐ minutes per litre

2. Velocity is the derivative of...

- ☐ acceleration
- ☐ position
- ☐ speed
- ☐ time

3. $C(x)$ is cost in dollars for x items. The units of the marginal cost $C'(x)$ are...

- ☐ dollars
- ☐ dollars per item
- ☐ items
- ☐ items per dollar

4. A related-rates problem links the rates of quantities that change with respect to...

- ☐ each other only
- ☐ time
- ☐ temperature
- ☐ nothing

5. With $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$, $r = 5$, and $\frac{dr}{dt} = 2$, find $\frac{dV}{dt}$ as a multiple of π (enter the number before π).

6. The linearization of f at a is $L(x) =$

- ☐ $f(a) + f'(a)(x - a)$
- ☐ $f'(a) + f(a)(x - a)$
- ☐ $f(a) \cdot f'(a)$
- ☐ $f(x) - f(a)$

7. L'Hospital's Rule applies to **which** indeterminate forms?

- ☐ $\frac{0}{0}$
- ☐ $\frac{\infty}{\infty}$
- ☐ $\frac{5}{2}$
- ☐ $\frac{3}{0}$

Tick every correct option.

8. For $v(t) = 3t^2 - 12t + 9$, the object is at rest when $v = 0$. One such time is $t = 1$; find the **other** time.

9. For $V(t) = 100 - 4t^2$, the rate is $V'(t) = -8t$. Find $V'(3)$ (litres per minute).

10. Using $L(x) = 2 + \frac{1}{4}(x - 4)$, estimate $\sqrt{4.1} = L(4.1)$.

AP Calculus AB

1. **litres per minute**

Output per input: litres per minute.

2. **position**

$v = s'$: velocity is the derivative of position.

3. **dollars per item**

Output per input: dollars per item.

4. **time**

The variables are functions of time, linked via the chain rule.

5. **200**

$$4\pi(25)(2) = 200\pi.$$

6. $f(a) + f'(a)(x - a)$

Point-slope of the tangent: $f(a) + f'(a)(x - a)$.

7. $\frac{0}{0}$; $\frac{\infty}{\infty}$

Only $\frac{0}{0}$ and $\frac{\infty}{\infty}$. $\frac{5}{2}$ is already determinate; $\frac{3}{0}$ is an asymptote, not indeterminate.

8. **3**

$$3(t^2 - 4t + 3) = 3(t - 1)(t - 3) = 0 \Rightarrow t = 1 \text{ or } t = 3.$$

9. **-24**

$$V'(3) = -8(3) = -24; \text{ draining at } 24 \text{ L/min.}$$

10. **2.025**

$$2 + \frac{1}{4}(0.1) = 2.025.$$