AP Calculus AB

AP readers award points for justification, units and interpretation as well as for the calculus. These solutions are written the way a scoring response reads: the setup shown in standard notation, the theorem's hypotheses stated before it is used, and every answer interpreted in context with units. 评分不仅看计算，还看依据、单位与情境解释。

Rates and interpretation

Water flows into a tank at $R(t) = 20 + 6\sin(t/3)$ litres per minute for $0 \le t \le 12$. Find the total volume added over the interval and interpret it. [3]

The total volume added is the integral of the rate: $\int_0^{12} \left(20 + 6\sin\frac{t}{3}\right) dt = 268.6$ litres. Interpretation: over the first 12 minutes, 268.6 litres of water flow into the tank.

Write the integral in standard notation before giving the calculator's value. Calculator syntax alone does not earn the setup point.

The integral of a rate is the NET CHANGE, not the amount present – if a starting volume were given it would have to be added.

The interpretation names the quantity, the interval and the units. All three are required.

For the same $R(t)$, find the time at which the flow rate is greatest, and justify your answer. [3]

$R'(t) = 2\cos(t/3)$, which is zero when $\cos(t/3) = 0$, so $t/3 = \pi/2$ and $t = 3\pi/2 \approx 4.712$. Since R' changes from positive to negative there, R has a maximum at $t \approx 4.712$ minutes.

The JUSTIFICATION is the sign change in R', not the fact that $R' = 0$. A critical point alone is not a maximum.

Endpoints must also be considered on a closed interval; here $R(4.712) = 26$ exceeds both endpoint values.

Accumulation and the Fundamental Theorem

Let $g(x) = \int_1^{x^2} \ln t \, dt$ for $x > 1$. Find $g'(x)$. [3]

By the Fundamental Theorem of Calculus with the chain rule, $g'(x) = \ln(x^2) \cdot \frac{d}{dx}(x^2) = 2x\ln(x^2)$.

The upper limit is a FUNCTION of x, so the chain rule applies. Omitting the factor $2x$ is the standard error and loses two of three.

Naming the theorem is worth doing; readers look for it.

Applying a theorem

A differentiable function f satisfies $f(2) = 5$ and $f(6) = 13$. Show that $f'(c) = 2$ for some c in $(2, 6)$. [3]

f is differentiable on $(2, 6)$ and therefore continuous on $[2, 6]$, so the Mean Value Theorem applies. It guarantees a c in $(2, 6)$ with $f'(c) = \dfrac{f(6) - f(2)}{6 - 2} = \dfrac{13 - 5}{4} = 2$.

State that the hypotheses hold BEFORE invoking the theorem. That sentence is the point; the arithmetic is the easy part.

Name the theorem explicitly. 'There must be a point where the slope is 2' asserts the conclusion without the justification.

The region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$ is rotated about the x-axis. Find the volume. [3]

Using discs perpendicular to the axis of rotation, $V = \pi \int_0^4 (\sqrt{x})^2 dx = \pi \int_0^4 x \, dx = \pi \left[\frac{x^2}{2}\right]_0^4 = \pi \cdot 8 = 8\pi$.

Write the integral with π and the squared radius before evaluating – the setup carries most of the marks.

The radius is the distance from the axis to the curve. Forgetting to square it is the commonest error in volume questions.