AP Calculus AB

Revision Sheet · 考前速记

Learn these by heart before the exam. 考试前把这些内容背熟。

Limits and Continuity

$\lim_{x\to c} f(x) = L \iff \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$ 极限存在的条件

$\lim_{x\to 0} \dfrac{\sin x}{x} = 1, \quad \lim_{x\to 0} \dfrac{1-\cos x}{x} = 0$ 两个特殊极限

Continuous at c needs all three: $f(c)$ exists, $\lim_{x\to c} f(x)$ exists, and they are equal.

Indeterminate $\frac{0}{0}$: factor, rationalise, or use L'Hopital. Never just substitute and stop.

$\lim_{x\to c} \dfrac{f(x)}{g(x)} = \lim_{x\to c} \dfrac{f'(x)}{g'(x)}$ only for $\frac{0}{0}$ or $\frac{\infty}{\infty}$; verify the form FIRST 洛必达法则

IVT: f continuous on $[a,b]$ means it takes every value between $f(a)$ and $f(b)$. Cite continuity explicitly.

Differentiation

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ 导数定义

$(x^n)' = nx^{n-1}, \quad (e^x)' = e^x, \quad (\ln x)' = \dfrac{1}{x}, \quad (a^x)' = a^x \ln a$ 基本导数

$(\sin x)' = \cos x,\ (\cos x)' = -\sin x,\ (\tan x)' = \sec^2 x$ 三角函数导数

$(uv)' = u'v + uv', \quad \left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$ 乘积与商法则

$\dfrac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$ 链式法则

$(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}, \quad (\arctan x)' = \dfrac{1}{1+x^2}$ 反三角函数导数

$\left(f^{-1}\right)'(a) = \dfrac{1}{f'(f^{-1}(a))}$ 反函数导数

Implicit: differentiate both sides in x, attach $\frac{dy}{dx}$ by the chain rule to every y, then solve.

Contextual Applications

Motion: $v(t) = x'(t)$, $a(t) = v'(t)$. Speed is $|v|$; speed rises when v and a share a sign.

total distance $= \int_a^b |v(t)|\, dt$, displacement $= \int_a^b v(t)\, dt$ 路程与位移

Related rates: write the relation, differentiate in t, THEN substitute the instant's values. Never substitute first.

$L(x) = f(a) + f'(a)(x-a)$ over-estimates when the curve is concave down 线性近似

Analytical Applications

$f' > 0$ increasing, $f' < 0$ decreasing. $f'' > 0$ concave up, $f'' < 0$ concave down.

Critical point: $f' = 0$ or undefined. It is a MAXIMUM if f' changes $+$ to $-$; say that in words.

Inflection point: f'' CHANGES SIGN. $f'' = 0$ alone is not enough.

$f'(c) = \dfrac{f(b)-f(a)}{b-a}$ needs continuous on $[a,b]$ and differentiable on (a,b); state both 中值定理

Candidates test for a closed interval: check every critical point AND both endpoints.

Integration and Accumulation

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C\ (n \neq -1), \quad \int \dfrac{1}{x} dx = \ln|x| + C$ 基本积分

$\int e^x dx = e^x + C, \quad \int \sin x\, dx = -\cos x + C, \quad \int \sec^2 x\, dx = \tan x + C$ 常用积分

$\dfrac{d}{dx} \int_a^{g(x)} f(t)\, dt = f(g(x))\, g'(x)$ 微积分基本定理 (第一部分)

$\int_a^b f'(x)\, dx = f(b) - f(a)$ 微积分基本定理 (第二部分)

$f_{\text{avg}} = \dfrac{1}{b-a} \int_a^b f(x)\, dx$ 函数平均值

u-substitution: change the LIMITS too, or convert back to x before evaluating. Do not mix.

Riemann sums: left under-estimates an increasing function, right over-estimates it. Trapezoid over-estimates when concave up.

Differential Equations

Separable: get all y with dy and all x with dx, integrate both sides, add ONE $+C$, then use the initial condition.

$\dfrac{dy}{dt} = ky \ \Rightarrow\ y = y_0 e^{kt}$ 指数增长与衰减

Slope field: at each point the segment has slope $\frac{dy}{dx}$. Follow the segments to sketch a solution curve.

Applications of Integration

$A = \int_a^b [f(x) - g(x)]\, dx$ top minus bottom; split where they cross 两曲线间的面积

$V = \pi \int_a^b \left[R(x)^2 - r(x)^2\right] dx$ 圆盘/圆环法体积

$V = \int_a^b A(x)\, dx$ square cross-section $A = s^2$; semicircle $A = \frac{\pi}{2} r^2$ 已知截面的体积

Accumulation: $f(b) = f(a) + \int_a^b f'(x) dx$. Read this as 'start plus the change'.

Exam technique

Justify in words, with the theorem named: 'by the MVT, since f is continuous on $[a,b]$ and differentiable on (a,b)'.

Calculator sections: show the SETUP integral or equation. A bare number earns almost nothing.

Keep 3 decimal places and units. $\frac{dy}{dx}$ answers usually need units of y per unit of x.